GRAVITATION

Introduction

Newton at the age of twenty-three is said to have seen an apple falling down from tree in his orchid. This was the year 1665. He started thinking about the role of earth's attraction in the motion of moon and other heavenly bodies.

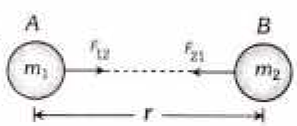
By comparing the acceleration due to gravity due to earth with the acceleration required to keep the moon in its orbit around the earth, he was able to arrive at the Basic Law of Gravitation.

Newton's law of Gravitation

Newton's law of gravitation states that every body in this universe attracts every other body with a force, which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres. The direction of the force is along the line joining the particles *i.e.*, it is a central force.

Thus, the magnitude of the gravitational force F that two particles of masses m_1 and m_2 are separated by a distance r exert on each other is given by $F \propto \frac{m_1 m_2}{r^2}$

or $F = G\frac{m_1 m_2}{r^2}$



Vector form : According to Newton's law of gravitation

$$\vec{F}_{12} = \frac{-Gm_1m_2}{r^2}\hat{r}_{21} = \frac{-Gm_1m_2}{r^3}\vec{r}_{21} = \frac{-Gm_1m_2}{|\vec{r}_{21}|^3}\vec{r}_{21}$$

Here negative sign indicates that the direction of $\vec{F}_{12}$ is opposite to that of $\hat{r}_{21}$.

Similarly $\vec{F}_{21} = \frac{-Gm_1m_2}{r^2}\hat{r}_{12} = \frac{-Gm_1m_2}{r^3}\vec{r}_{12} = \frac{-Gm_1m_2}{|\vec{r}_{12}|^3}\vec{r}_{12}$

$$= \frac{Gm_1m_2}{r^2}\hat{r}_{21} \quad [\because \hat{r}_{12} = -\hat{r}_{21}]$$

$\therefore$ It is clear that $\vec{F}_{12} = -\vec{F}_{21}$ which is Newton's third law of motion *i.e.*, $\vec{F}_{12} + \vec{F}_{21} = 0$. Hence this force is a conservative force.

Here G is constant of proportionality which is called 'Universal gravitational constant'.

If $m_1 = m_2$ and $r = 1$ then $G = F$

i.e. universal gravitational constant is equal to the force of attraction between two bodies each of unit mass whose centres are placed unit distance apart.

(i) The value of G in the laboratory was first determined by Cavendish using the torsional balance.

(ii) The value of G is 6.67×10^{-11} $N\text{-}m^2\ kg^{-2}$ in S.I. and 6.67×10^{-8} $dyne\text{-}cm^2\text{-}g^{-2}$ in C.G.S. system.

(iii) Dimensional formula $[M^{-1}L^3T^{-2}]$.

(iv) The value of G does not depend upon the nature and size of the bodies.

(v) It also does not depend upon the nature of the medium between the two bodies.

(vi) As G is very small, hence gravitational forces are very small, unless one (or both) of the mass is huge.

Properties of Gravitational Force

(1) It is always attractive in nature while electric and magnetic force can be attractive or repulsive.

(2) It is independent of the medium between the particles while electric and magnetic force depends on the nature of the medium between the particles.

(3) It holds good over a wide range of distances. It is found true for interplanetary to inter atomic distances.

(4) It is a central force *i.e.* acts along the line joining the centres of two interacting bodies.

(5) It is a two-body interaction *i.e.* gravitational force between two particles is independent of the presence or absence of other particles; so the principle of superposition is valid *i.e.* force on a particle due to number of particles is the resultant of forces due to individual particles *i.e.* $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ...$

While nuclear force is many body interaction

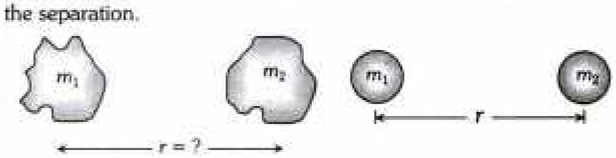
(6) It is the weakest force in nature.

(7) The ratio of gravitational force to electrostatic force between two electrons is of the order of 10^{-43} .

(8) It is a conservative force *i.e.* work done by it is path independent or work done in moving a particle round a closed path under the action of gravitational force is zero.

(9) It is an action reaction pair *i.e.* the force with which one body (say earth) attracts the second body (say moon) is equal to the force with which moon attracts the earth. This is in accordance with Newton's third law of motion.

Note : ❑ The law of gravitation is stated for two point masses, therefore for any two arbitrary finite size bodies, as shown in the figure, it can not be applied as there is no unique value for the separation.



But if the two bodies are uniform spheres then the separation r may be taken as the distance between their centres because a sphere of uniform mass behaves as a point mass for any point lying outside it.

Acceleration due to Gravity

The force of attraction exerted by the earth on a body is called gravitational pull or gravity.

We know that when force acts on a body, it produces acceleration. Therefore, a body under the effect of gravitational pull must accelerate.

The acceleration produced in the motion of a body under the effect of gravity is called acceleration due to gravity, it is denoted by g.

Consider a body of mass m, lying on the surface of earth then gravitational force on the body is given by

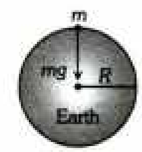
$$F = \frac{GMm}{R^2} \quad ...(i)$$

Where M = mass of the earth and R = radius of the earth.

If g is the acceleration due to gravity, then the force on the body due to earth is given by

Force = mass × acceleration

or $\quad F = mg \quad ...(ii)$



From (i) and (ii) we have $mg = \frac{GMm}{R^2}$

$$\therefore \; g = \frac{GM}{R^2} \quad ...(iii)$$

$$\Rightarrow g = \frac{G}{R^2}\left(\frac{4}{3}\pi R^3 \rho\right)$$

[As mass (M) = volume $(\frac{4}{3}\pi R^3)$ × density (ρ)]

$$\therefore \; g = \frac{4}{3}\pi\rho GR \quad ...(iv)$$

(i) From the expression $g = \frac{GM}{R^2} = \frac{4}{3}\pi\rho GR$ it is clear that its value depends upon the mass, radius and density of planet and it is independent of mass, shape and density of the body placed on the surface of the planet. *i.e.* a given planet (reference body) produces same acceleration in a light as well as heavy body.

(ii) The greater the value of (M/R^2) or ρR, greater will be value of g for that planet.

(iii) Acceleration due to gravity is a vector quantity and its direction is always towards the centre of the planet.

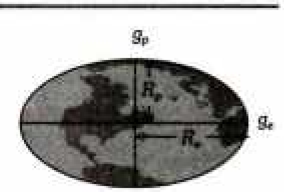
(iv) Dimension $[g] = [LT^{-2}]$

(v) It's average value is taken to be 9.8 m/s^2 or 981 cm/sec^2 or 32 $feet/s^2$, on the surface of the earth at mean sea level.

(vi) The value of acceleration due to gravity vary due to the following factors : (a) Shape of the earth, (b) Height above the earths surface, (c) Depth below the earths surface and (d) Axial rotation of the earth.

Variation in *g* due to Shape of Earth

Earth is elliptical in shape. It is flattened at the poles and bulged out at the equator. The equatorial radius is about 21 km longer than polar radius, from $g = \frac{GM}{R^2}$



At equator $g_e = \frac{GM}{R_e^2}$ ...(i)

At poles $g_p = \frac{GM}{R_p^2}$ ...(ii)

From (i) and (ii) $\frac{g_e}{g_p} = \frac{R_p^2}{R_e^2}$

Since $R_{equator} > R_{pole}$

$\therefore$ $g_{pole} > g_{equator}$ and $g_p = g_e + 0.018\ ms^{-2}$

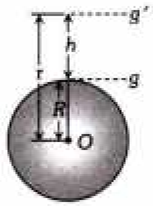
Therefore the weight of body increases as it is taken from equator to the pole.

Variation in *g* with Height

Acceleration due to gravity at the surface of the earth

$$g = \frac{GM}{R^2} \quad ...(i)$$

Acceleration due to gravity at height h from the surface of the earth



$$g' = \frac{GM}{(R+h)^2} \quad ...(ii)$$

From (i) and (ii) $g' = g\left(\frac{R}{R+h}\right)^2$ ...(iii)

$$= g\frac{R^2}{r^2} \quad ...(iv)$$

[As $r = R + h$]

(i) As we go above the surface of the earth, the value of g decreases because $g' \propto \frac{1}{r^2}$.

(ii) If $r = \infty$ then $g' = 0$, *i.e.*, at infinite distance from the earth, the value of g becomes zero.

(iii) If $h << R$ *i.e.*, height is negligible in comparison to the radius then from equation (iii) we get

$$g' = g\left(\frac{R}{R+h}\right)^2 = g\left(1+\frac{h}{R}\right)^{-2} = g\left[1-\frac{2h}{R}\right]$$

[As $h << R$]

(iv) If $h << R$ then decrease in the value of g with height :

Absolute decrease $\Delta g = g - g' = \frac{2hg}{R}$

Fractional decrease $\frac{\Delta g}{g} = \frac{g-g'}{g} = \frac{2h}{R}$

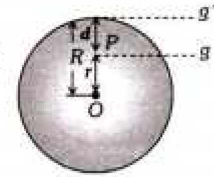
Percentage decrease $\frac{\Delta g}{g} \times 100\% = \frac{2h}{R} \times 100\%$

Variation in *g* With Depth

Acceleration due to gravity at the surface of the earth

$$g = \frac{GM}{R^2} = \frac{4}{3}\pi\rho GR \quad ...(i)$$

Acceleration due to gravity at depth d from the surface of the earth



$$g' = \frac{4}{3}\pi\rho G(R-d) \quad ...(ii)$$

From (i) and (ii) $g' = g\left[1-\frac{d}{R}\right]$

(i) The value of g decreases on going below the surface of the earth. From equation (ii) we get $g' \propto (R-d)$.

So it is clear that if d increases, the value of g decreases.

(ii) At the centre of earth $d = R$ $\therefore$ $g' = 0$, i.e., the acceleration due to gravity at the centre of earth becomes zero.

(iii) Decrease in the value of g with depth

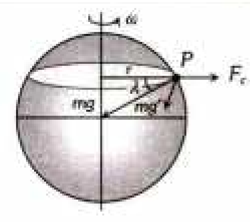
Absolute decrease $\Delta g = g - g' = \frac{dg}{R}$

Fractional decrease $\frac{\Delta g}{g} = \frac{g-g'}{g} = \frac{d}{R}$

Percentage decrease $\frac{\Delta g}{g} \times 100\% = \frac{d}{R} \times 100\%$

(iv) The rate of decrease of gravity outside the earth (if $h << R$) is double to that of inside the earth.

Variation in *g* due to Rotation of Earth



As the earth rotates, a body placed on its surface moves along the circular path and hence experiences centrifugal force, due to it, the apparent weight of the body decreases.

Since the magnitude of centrifugal force varies with the latitude of the place, therefore the apparent weight of the body varies with latitude due to variation in the magnitude of centrifugal force on the body.

If the body of mass m lying at point P, whose latitude is λ, then due to rotation of earth its apparent weight can be given by $m\vec{g}' = m\vec{g} + \vec{F}_c$

or $mg' = \sqrt{(mg)^2 + (F_c)^2 + 2mg\ F_c\ \cos(180-\lambda)}$

$\Rightarrow$ $mg' = \sqrt{(mg)^2 + (m\omega^2 R\cos\lambda)^2 + 2mg\ m\omega^2 R\cos\lambda\ (-\cos\lambda)}$

[As $F_c = m\omega^2 r = m\omega^2 R\cos\lambda$]

By solving we get $g' = g - \omega^2 R\cos^2\lambda$

Note : ❑ The latitude at a point on the surface of the earth is defined as the angle, which the line joining that point to the centre of earth makes with equatorial plane. It is denoted by λ.

❑ For the poles $\lambda = 90°$ and for equator $\lambda = 0°$

(i) Substituting $\lambda = 90^\circ$ in the above expression we get $g_{pole} = g - \omega^2 R\cos^2 90^\circ$

$\therefore\ g_{pole} = g$ ...(i)

i.e., there is no effect of rotational motion of the earth on the value of g at the poles.

(ii) Substituting $\lambda = 0^\circ$ in the above expression we get $g_{equator} = g - \omega^2 R\cos^2 0^\circ$

$\therefore\ g_{equator} = g - \omega^2 R$ ...(ii)

i.e., the effect of rotation of earth on the value of g at the equator is maximum.

From equation (i) and (ii)

$g_{pole} - g_{equator} = R\omega^2 = 0.034 m/s^2$

(iii) When a body of mass m is moved from the equator to the poles, its weight increases by an amount

$m(g_p - g_e) = m\omega^2 R$

(iv) Weightlessness due to rotation of earth : As we know that apparent weight of the body decreases due to rotation of earth. If ω is the angular velocity of rotation of earth for which a body at the equator will become weightless

$g' = g - \omega^2 R\cos^2\lambda$

$\Rightarrow 0 = g - \omega^2 R\cos^2 0^\circ$ [As $\lambda = 0^\circ$ for equator]

$\Rightarrow g - \omega^2 R = 0$

$\therefore\ \omega = \sqrt{\dfrac{g}{R}}$

or time period of rotation of earth $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{R}{g}}$

Substituting the value of $R = 6400\times 10^3 m$ and $g = 10 m/s^2$ we get

$\omega = \dfrac{1}{800} = 1.25\times 10^{-3}\dfrac{rad}{sec}$ and $T = 5026.5\ sec = 1.40\ hr.$

Note : ❑ This time is about $\dfrac{1}{17}$ times the present time period of earth. Therefore if earth starts rotating 17 times faster then all objects on equator will become weightless.

❑ If earth stops rotation about its own axis then at the equator the value of g increases by $\omega^2 R$ and consequently the weight of body lying there increases by $m\omega^2 R$.

❑ After considering the effect of rotation and elliptical shape of the earth, acceleration due to gravity at the poles and equator are related as

$g_p = g_e + 0.034 + 0.018 m/s^2 \quad \therefore g_p = g_e + 0.052 m/s^2$

Mass and Density of Earth

Newton's law of gravitation can be used to estimate the mass and density of the earth.

As we know $g = \dfrac{GM}{R^2}$, so we have $M = \dfrac{gR^2}{G}$

$\therefore\ M = \dfrac{9.8\times(6.4\times 10^6)^2}{6.67\times 10^{-11}} = 5.98\times 10^{24} kg \approx 10^{25} kg$

and as we know $g = \dfrac{4}{3}\pi\rho GR$, so we have $\rho = \dfrac{3g}{4\pi GR}$

$\therefore\ \rho = \dfrac{3\times 9.8}{4\times 3.14\times 6.67\times 10^{-11}\times 6.4\times 10^6} = 5478.4\ kg/m^3$

Inertial and Gravitational Masses

(1) **Inertial mass** : It is the mass of the material of the body, which measures its inertia.

If an external force F acts on a body of mass m_i, then according to Newton's second law of motion

$F = m_i a$ or $m_i = \dfrac{F}{a}$

Hence inertial mass of a body may be measured as the ratio of the magnitude of the external force applied on it to the magnitude of acceleration produced in its motion.

(i) It is the measure of ability of the body to oppose the production of acceleration in its motion by an external force.

(ii) Gravity has no effect on inertial mass of the body.

(iii) It is proportional to the quantity of matter contained in the body.

(iv) It is independent of size, shape and state of body.

(v) It does not depend on the temperature of body.

(vi) It is conserved when two bodies combine physically or chemically.

(vii) When a body moves with velocity v, its inertial mass is given by

$m = \dfrac{m_0}{\sqrt{1 - \dfrac{v^2}{c^2}}}$, where m_0 = rest mass of body, c = velocity of light in vacuum,

(2) **Gravitational Mass** : It is the mass of the material of body, which determines the gravitational pull acting upon it.

If M is the mass of the earth and R is the radius, then gravitational pull on a body of mass m_g is given by

$F = \dfrac{GMm_g}{R^2}$ or $m_g = \dfrac{F}{GM/R^2} = \dfrac{F}{I}$

Here m_g is the gravitational mass of the body, if $I = 1$ then $m_g = F$

Thus the gravitational mass of a body is defined as the gravitational pull experienced by the body in a gravitational field of unit intensity.

(3) **Comparison between inertial and gravitational mass**

(i) Both are measured in the same units.

(ii) Both are scalar.

(iii) Both do not depend on the shape and state of the body

(iv) Inertial mass is measured by applying Newton's second law of motion where as gravitational mass is measured by applying Newton's law of gravitation.

(v) Spring balance measure gravitational mass and physical balance measure inertial mass.

(4) **Comparison between mass and weight of the body**

Mass (*m*)	**Weight (*W*)**
It is a quantity of matter contained in a body.	It is the attractive force exerted by earth on any body.
Its value does not change with *g*	Its value changes with *g*.
Its value can never be zero for any material particle.	At infinity and at the centre of earth its value is zero.
Its unit is kilogram and its dimension is $[M]$.	Its unit is Newton or *kg-wt* and dimension are $[MLT^{-2}]$
It is determined by a physical balance.	It is determined by a spring balance.
It is a scalar quantity.	It is a vector quantity.

Gravitational Field

The space surrounding a material body in which gravitational force of attraction can be experienced is called its gravitational field.

Gravitational field intensity : The intensity of the gravitational field of a material body at any point in its field is defined as the force experienced by a unit mass (test mass) placed at that point, provided the unit mass (test mass) itself does not produce any change in the field of the body.

So if a test mass m at a point in a gravitational field experiences a force $\vec{F}$ then

$$\vec{I} = \frac{\vec{F}}{m}$$

(i) It is a vector quantity and is always directed towards the centre of gravity of body whose gravitational field is considered.

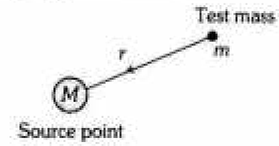
(ii) Units : *Newton/kg* or m/s^2

(iii) Dimension : $[M^0LT^{-2}]$

(iv) If the field is produced by a point mass M and the test mass m is at a distance r from it then by Newton's law of gravitation $F = \frac{GMm}{r^2}$, then intensity of gravitational field

$$I = \frac{F}{m} = \frac{GMm/r^2}{m}$$

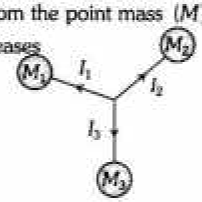
$$\therefore \; I = \frac{GM}{r^2}$$



(v) As the distance (r) of test mass from the point mass (M), increases, intensity of gravitational field decreases

$$I = \frac{GM}{r^2};$$

$$\therefore \; I \propto \frac{1}{r^2}$$



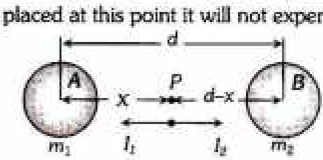
(vi) Intensity of gravitational field $I = 0$, when $r = \infty$.

(vii) Intensity at a given point (P) due to the combined effect of different point masses can be calculated by vector sum of different intensities

$$\vec{I}_{net} = \vec{I}_1 + \vec{I}_2 + \vec{I}_3 + ...$$

(viii) Point of zero intensity : If two bodies A and B of different masses m_1 and m_2 are d distance apart.

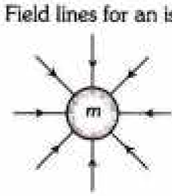
Let P be the point of zero intensity *i.e.*, the intensity at this point is equal and opposite due to two bodies A and B and if any test mass placed at this point it will not experience any force.



For point P, $\vec{I}_1 + \vec{I}_2 = 0 \;\Rightarrow\; \frac{-Gm_1}{x^2} + \frac{Gm_2}{(d-x)^2} = 0$

By solving $x = \frac{\sqrt{m_1}\, d}{\sqrt{m_1} + \sqrt{m_2}}$ and $(d - x) = \frac{\sqrt{m_2}\, d}{\sqrt{m_1} + \sqrt{m_2}}$

(ix) Gravitational field line is a line, straight or curved such that a unit mass placed in the field of another mass would always move along this line. Field lines for an isolated mass m are radially inwards.

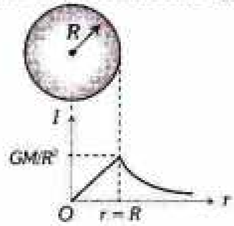


(x) As $I = \frac{GM}{r^2}$ and also $g = \frac{GM}{R^2}$ $\therefore$ $I = g$

Thus the intensity of gravitational field at a point in the field is equal to acceleration of test mass placed at that point.

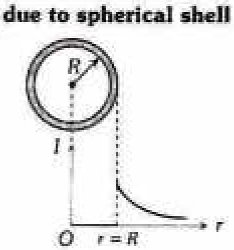
Gravitational Field Intensity for Different Bodies

(1) **Intensity due to uniform solid sphere**



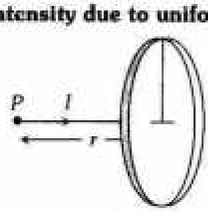
Outside the surface $r > R$	**On the surface $r = R$**	**Inside the surface $r < R$**
$I = \frac{GM}{r^2}$	$I = \frac{GM}{R^2}$	$I = \frac{GMr}{R^3}$

(2) **Intensity due to spherical shell**



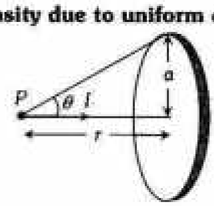
Outside the surface $r > R$	**On the surface $r = R$**	**Inside the surface $r < R$**
$I = \frac{GM}{r^2}$	$I = \frac{GM}{R^2}$	$I = 0$

(3) **Intensity due to uniform circular ring**



At a point on its axis	**At the centre of the ring**
$I = \frac{GMr}{(a^2 + r^2)^{3/2}}$	$I = 0$

(4) **Intensity due to uniform disc**



At a point on its axis	**At the centre of the disc**
$I = \frac{2GMr}{a^2}\left[\frac{1}{r} - \frac{1}{\sqrt{r^2 + a^2}}\right]$ or $I = \frac{2GM}{a^2}(1 - \cos\theta)$	$I = 0$

Gravitational Potential

At a point in a gravitational field potential V is defined as negative of work done per unit mass in shifting a test mass from some reference point (usually at infinity) to the given point *i.e.*,

$$V = -\frac{W}{m} = -\int \frac{\vec{F}.d\vec{r}}{m} = -\int \vec{I}.d\vec{r} \qquad \left[\text{As } \frac{F}{m} = I\right]$$

$$\therefore \; I = -\frac{dV}{dr}$$

i.e., negative gradient of potential gives intensity of field or potential is a scalar function of position whose space derivative gives intensity. Negative sign indicates that the direction of intensity is in the direction where the potential decreases.

(i) It is a scalar quantity because it is defined as work done per unit mass.

(ii) Unit : *Joule/kg* or m^2/sec^2

(iii) Dimension : $[M^0L^2T^{-2}]$

(iv) If the field is produced by a point mass then

$$V = -\int I\, dr = -\int\left(-\frac{GM}{r^2}\right)dr \qquad \left[\text{As } I = -\frac{GM}{r^2}\right]$$

$$\therefore \; V = -\frac{GM}{r} + c \qquad [\text{Here } c = \text{constant of integration}]$$

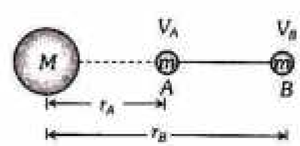
Assuming reference point at ∞ and potential to be zero there we get

$$0 = -\frac{GM}{\infty} + c \Rightarrow c = 0$$

$\therefore$ Gravitational potential $V = -\frac{GM}{r}$

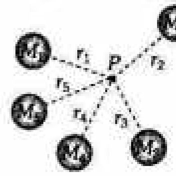
At $r = \infty$, $V = 0 = V_{max}$

(v) Gravitational potential difference : It is defined as the work done to move a unit mass from one point to the other in the gravitational field. The gravitational potential difference in bringing unit test mass m from point A to point B under the gravitational influence of source mass M is



$$\Delta V = V_B - V_A = \frac{W_{A \to B}}{m} = -GM\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

(vi) Potential due to large numbers of particle is given by scalar addition of all the potentials.



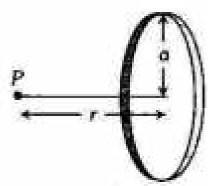
$$V = V_1 + V_2 + V_3 + \ldots\ldots\ldots$$

$$= -\frac{GM}{r_1} - \frac{GM}{r_2} - \frac{GM}{r_3} \ldots\ldots\ldots$$

$$= -G\sum_{i=1}^{i=n}\frac{M_i}{r_i}$$

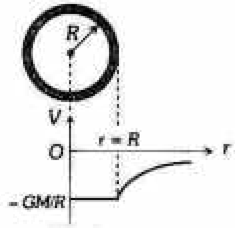
Gravitational Potential for Different Bodies

(1) **Potential due to uniform ring**



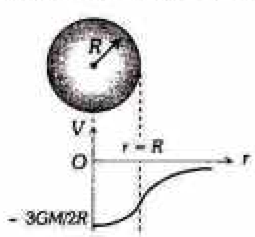
At a point on its axis	At the centre
$V = -\frac{GM}{\sqrt{a^2+r^2}}$	$V = -\frac{GM}{a}$

(2) **Potential due to spherical shell**



Outside the surface $r > R$	On the surface $r = R$	Inside the surface $r < R$
$V = \frac{-GM}{r}$	$V = \frac{-GM}{R}$	$V = \frac{-GM}{R}$

(3) **Potential due to uniform solid sphere**

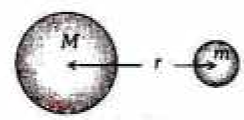


Outside the surface $r > R$	On the surface $r = R$	Inside the surface $r < R$
$V = \frac{-GM}{r}$	$V_{surface} = \frac{-GM}{R}$	$V = \frac{-GM}{2R}\left[3 - \left(\frac{r}{R}\right)^2\right]$ at the centre ($r = 0$) $V_{centre} = \frac{-3}{2}\frac{GM}{R}$ (max.) $V_{centre} = \frac{3}{2}V_{surface}$

Gravitational Potential Energy

The gravitational potential energy of a body at a point is defined as the amount of work done in bringing the body from infinity to that point against the gravitational force.

$$W = \int_{\infty}^{r}\frac{GMm}{x^2}dx = -GMm\left[\frac{1}{x}\right]_{\infty}^{r}$$



$$W = -\frac{GMm}{r}$$

This work done is stored inside the body as its gravitational potential energy

$$\therefore U = -\frac{GMm}{r}$$

(i) Potential energy is a scalar quantity.

(ii) *Unit* : Joule

(iii) *Dimension* : $[ML^2T^{-2}]$

(iv) Gravitational potential energy is always negative in the gravitational field because the force is always attractive in nature.

(v) As the distance r increases, the gravitational potential energy becomes less negative i.e., it increases.

(vi) If $r = \infty$ then it becomes zero (maximum)

(vii) In case of discrete distribution of masses

Gravitational potential energy

$$U = \sum u_i = -\left[\frac{Gm_1m_2}{r_{12}} + \frac{Gm_2m_3}{r_{23}} + \ldots\ldots\right]$$

(viii) If the body of mass m is moved from a point at a distance r_1 to a point at distance $r_2 (r_1 > r_2)$ then change in potential energy $\Delta U = \int_{r_1}^{r_2}\frac{GMm}{x^2}dx = -GMm\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$

$$\text{or } \Delta U = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

As r_1 is greater than r_2, the change in potential energy of the body will be negative. It means that if a body is brought closer to earth it's potential energy decreases.

(ix) Relation between gravitational potential energy and potential $U = -\frac{GMm}{r} = m\left[\frac{-GM}{r}\right]$

$\therefore \; U = mV$

(x) Gravitational potential energy at the centre of earth relative to infinity.

$$U_{centre} = mV_{centre} = m\left(-\frac{3}{2}\frac{GM}{R}\right) = -\frac{3}{2}\frac{GMm}{R}$$

(xi) Gravitational potential energy of a body at height h from the earth surface is given by

$$U_h = -\frac{GMm}{R+h} = -\frac{gR^2m}{R+h} \equiv -\frac{mgR}{1+\frac{h}{R}}$$

Work Done Against Gravity

If the body of mass m is moved from the surface of earth to a point at distance h above the surface of earth, then change in potential energy or work done against gravity will be

$$W = \Delta U = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$\Rightarrow W = GMm\left[\frac{1}{R} - \frac{1}{R+h}\right] \quad [\text{As } r_1 = R \text{ and } r_2 = R+h]$$

$$\Rightarrow W = \frac{GMmh}{R^2\left(1+\frac{h}{R}\right)} = \frac{mgh}{1+\frac{h}{R}} \quad [\text{As } \frac{GM}{R^2} = g]$$

(i) When the distance h is not negligible and is comparable to radius of the earth, then we will use above formula.

(ii) If $h = nR$ then $W = mgR\left(\frac{n}{n+1}\right)$

(iii) If $h = R$ then $W = \frac{1}{2}mgR$

(iv) If h is very small as compared to radius of the earth then term h/R can be neglected

From $W = \frac{mgh}{1+h/R} = mgh$ $\quad \left[\text{As } \frac{h}{R} \to 0\right]$

Escape Velocity

The minimum velocity with which a body must be projected up so as to enable it to just overcome the gravitational pull, is known as escape velocity.

The work done to displace a body from the surface of earth ($r = R$) to infinity ($r = \infty$) is

$$W = \int_R^{\infty} \frac{GMm}{x^2}dx = -GMm\left[\frac{1}{\infty} - \frac{1}{R}\right]$$

$$\Rightarrow W = \frac{GMm}{R}$$

This work required to project the body so as to escape the gravitational pull is performed on the body by providing an equal amount of kinetic energy to it at the surface of the earth.

If v_e is the required escape velocity, then kinetic energy which should be given to the body is $\frac{1}{2}mv_e^2$

$$\therefore \; \frac{1}{2}mv_e^2 = \frac{GMm}{R} \Rightarrow v_e = \sqrt{\frac{2GM}{R}}$$

$$\Rightarrow \; v_e = \sqrt{2gR} \qquad [\text{As } GM = gR^2]$$

$$\text{or } v_e = \sqrt{2 \times \frac{4}{3}\pi\rho GR \times R} \Rightarrow v_e = R\sqrt{\frac{8}{3}\pi G\rho}$$

$$[\text{As } g = \frac{4}{3}\pi\rho GR]$$

(i) Escape velocity is independent of the mass and direction of projection of the body.

(ii) Escape velocity depends on the reference body. Greater the value of (M/R) or (gR) for a planet, greater will be escape velocity.

(iii) For the earth as $g = 9.8 m/s^2$ and $R = 6400 km$

$$\therefore v_e = \sqrt{2 \times 9.8 \times 6.4 \times 10^6} = 11.2 km/sec$$

(iv) A planet will have atmosphere if the velocity of molecule in its atmosphere $\left[v_{rms} = \sqrt{\frac{3RT}{M}}\right]$ is lesser than escape velocity. This is why earth has atmosphere (as at earth $v_{rms} < v_e$) while moon has no atmosphere (as at moon $v_{rms} > v_e$)

(v) If a body projected with velocity lesser than escape velocity ($v < v_e$), it will reach a certain maximum height and then may either move in an orbit around the planet or may fall down back to the planet.

(vi) Maximum height attained by body : Let a projection velocity of body (mass m) is v, so that it attains a maximum height h. At maximum height, the velocity of particle is zero, so kinetic energy is zero.

By the law of conservation of energy

Total energy at surface = Total energy at height h.

$$\Rightarrow -\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{R+h} + 0$$

$$\Rightarrow \frac{v^2}{2} = GM\left[\frac{1}{R} - \frac{1}{R+h}\right] = \frac{GMh}{R(R+h)}$$

$$\Rightarrow \frac{2GM}{v^2R} = \frac{R+h}{h} = 1 + \frac{R}{h}$$

$$\Rightarrow h = \frac{R}{\left(\frac{2GM}{v^2R} - 1\right)} = \frac{R}{\frac{v_e^2}{v^2} - 1} = R\left[\frac{v^2}{v_e^2 - v^2}\right]$$

$$[\text{As } v_e = \sqrt{\frac{2GM}{R}} \;\therefore\; \frac{2GM}{R} = v_e^2]$$

(vii) If a body is projected with velocity greater than escape velocity $(v > v_e)$ then by conservation of energy.

Total energy at surface = Total energy at infinite

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = \frac{1}{2}m(v')^2 + 0$$

i.e., $(v')^2 = v^2 - \frac{2GM}{R} \Rightarrow v'^2 = v^2 - v_e^2$ [As $\frac{2GM}{R} = v_e^2$]

$$\therefore \; v' = \sqrt{v^2 - v_e^2}$$

i.e, the body will move in interplanetary or inter stellar space with velocity $\sqrt{v^2 - v_e^2}$.

(viii) Energy to be given to a stationary object on the surface of earth so that its total energy becomes zero, is called escape energy.

Total energy at the surface of the earth $= KE + PE = 0 - \frac{GMm}{R}$

$$\therefore \text{ Escape energy } = \frac{GMm}{R}$$

(ix) If the escape velocity of a body is equal to the velocity of light then from such bodies nothing can escape, not even light. Such bodies are called black holes.

The radius of a black hole is given as

$$R = \frac{2GM}{C^2}$$

[As $C = \sqrt{\frac{2GM}{R}}$, where C is the velocity of light]

Kepler's Laws of Planetary Motion

Planets are large natural bodies rotating around a star in definite orbits. The planetary system of the star sun called solar system consists of nine planets, viz., Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune and Pluto. Out of these planets Mercury is the smallest and closest to the sun and so hottest. Jupiter is largest and has maximum moons (12). Venus is closest to Earth and brightest. Kepler after a life time study, worked out three empirical laws which govern the motion of these planets and are known as *Kepler's laws of planetary motion*. These are,

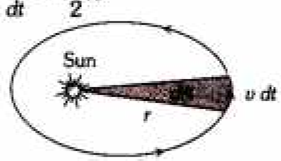
(1) **The law of Orbits** : Every planet moves around the sun in an elliptical orbit with sun at one of the foci.

(2) **The law of Area** : The line joining the sun to the planet sweeps out equal areas in equal interval of time. i.e. aerial velocity is constant. According to this law planet will move slowly when it is farthest from sun and more rapidly when it is nearest to sun. It is similar to law of conservation of angular momentum.

Aerial velocity $= \frac{dA}{dt} = \frac{1}{2}\frac{r(vdt)}{dt} = \frac{1}{2}rv$

$$\therefore \; \frac{dA}{dt} = \frac{L}{2m}$$

[As $L = mvr$; $rv = \frac{L}{m}$]

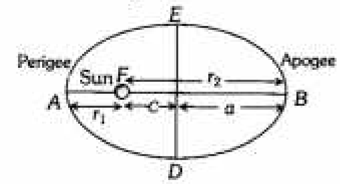


(3) **The law of periods** : The square of period of revolution (T) of any planet around sun is directly proportional to the cube of the semi-major axis of the orbit.

$$T^2 \propto a^3 \text{ or } T^2 \propto \left(\frac{r_1 + r_2}{2}\right)^3$$

Proof : From the figure $AB = AF + FB$

$$2a = r_1 + r_2 \quad \therefore a = \frac{r_1 + r_2}{2}$$



where a = semi-major axis

r_1 = Shortest distance of planet from sun (perigee).

r_2 = Largest distance of planet from sun (apogee).

Important data

Planet	Semi-major axis a (10^{10} metre)	Period T(year)	T^2/a^3 (10^{-34} year^2/metre^3)
Mercury	5.79	0.241	2.99
Venus	10.8	0.615	3.00
Earth	15.0	1.00	2.96
Mars	22.8	1.88	2.98
Jupiter	77.8	11.9	3.01
Saturn	143	29.5	2.98
Uranus	287	84.0	2.98
Neptune	450	165	2.99
Pluto	590	248	2.99

Note : ☐ Kepler's laws are valid for satellites also.

Velocity of a Planet in Terms of Eccentricity

Applying the law of conservation of angular momentum at perigee and apogee

$$mv_p r_p = mv_a r_a$$

$$\Rightarrow \frac{v_p}{v_a} = \frac{r_a}{r_p} = \frac{a+c}{a-c} = \frac{1+e}{1-e}$$

[As $r_p = a - c$, $r_a = a + c$ and eccentricity $e = \frac{c}{a}$]

Applying the conservation of mechanical energy at perigee and apogee

$$\frac{1}{2}mv_p^2 - \frac{GMm}{r_p} = \frac{1}{2}mv_a^2 - \frac{GMm}{r_a}$$

$$\Rightarrow v_p^2 - v_a^2 = 2GM\left[\frac{1}{r_p} - \frac{1}{r_a}\right]$$

$$\Rightarrow v_a^2\left[\frac{r_a^2 - r_p^2}{r_p^2}\right] = 2GM\left[\frac{r_a - r_p}{r_a r_p}\right] \quad \left[\text{As } v_p = \frac{v_a r_a}{r_p}\right]$$

$$\Rightarrow v_a^2 = \frac{2GM}{r_a + r_p}\left[\frac{r_p}{r_a}\right] \Rightarrow v_a^2 = \frac{2GM}{a}\left(\frac{a-c}{a+c}\right) = \frac{GM}{a}\left(\frac{1-e}{1+e}\right)$$

Thus the speeds of planet at apogee and perigee are

$$v_a = \sqrt{\frac{GM}{a}\left(\frac{1-e}{1+e}\right)},$$

$$v_p = \sqrt{\frac{GM}{a}\left(\frac{1+e}{1-e}\right)}$$

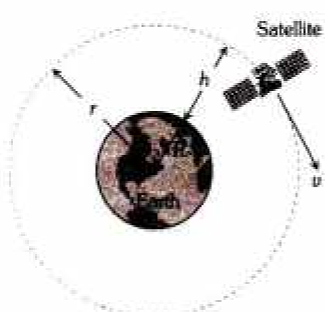
Note: ❑ The gravitational force is a central force so torque on planet relative to sun is always zero, hence angular momentum of a planet or satellite is always constant irrespective of shape of orbit.

Some Properties of the Planet

	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune	Pluto
Mean distance from sun, 10^6 *km*	57.9	108	150	228	778	1430	2870	4500	5900
Period of revolution, *year*	0.241	0.615	1.00	1.88	11.9	29.5	84.0	165	248
Orbital speed, *km/s*	47.9	35.0	29.8	24.1	13.1	9.64	6.81	5.43	4.74
Equatiorial diameter, *km*	4880	12100	12800	6790	143000	120000	51800	49500	2300
Mass (Earth =1)	0.0558	0.815	1.000	0.107	318	95.1	14.5	17.2	0.002
Density (Water =1)	5.60	5.20	5.52	3.95	1.31	0.704	1.21	1.67	2.03
Surface value of *g*, m/s^2	3.78	8.60	9.78	3.72	22.9	9.05	7.77	11.0	0.5
Escape velocity, *km/s*	4.3	10.3	11.2	5.0	59.5	35.6	21.2	23.6	1.1
Known satellites	0	0	1	2	16+ring	18+rings	17+rings	8+rings	1

Orbital Velocity of Satellite

Satellites are natural or artificial bodies describing orbit around a planet under its gravitational attraction. Moon is a natural satellite while INSAT-1B is an artificial satellite of earth. Condition for establishment of artificial satellite is that the centre of orbit of satellite must coincide with centre of earth or satellite must move around great circle of earth.



Orbital velocity of a satellite is the velocity required to put the satellite into its orbit around the earth.

For revolution of satellite around the earth, the gravitational pull provides the required centripetal force.

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$\Rightarrow v = \sqrt{\frac{GM}{r}}$$

$$v = \sqrt{\frac{gR^2}{R+h}} = R\sqrt{\frac{g}{R+h}}$$

[As $GM = gR^2$ and $r = R + h$]

(i) Orbital velocity is independent of the mass of the orbiting body and is always along the tangent of the orbit *i.e.*, satellites of different masses have same orbital velocity, if they are in the same orbit.

(ii) Orbital velocity depends on the mass of central body and radius of orbit.

(iii) For a given planet, greater the radius of orbit, lesser will be the orbital velocity of the satellite $\left(v \propto 1/\sqrt{r}\right)$.

(iv) Orbital velocity of the satellite when it revolves very close to the surface of the planet

$$v=\sqrt{\frac{GM}{r}}=\sqrt{\frac{GM}{R+h}} \quad \therefore\ v=\sqrt{\frac{GM}{R}}=\sqrt{gR}$$

[As $h=0$ and $GM=gR^2$]

For the earth $v=\sqrt{9.8\times 6.4\times 10^6}$ $=7.9\,km/s\approx 8\,km/sec$

(v) Close to the surface of planet $v=\sqrt{\frac{GM}{R}}$

[As $v_e=\sqrt{\frac{2GM}{R}}$]

$\therefore\ v=\frac{v_e}{\sqrt{2}}$ i.e., $v_{escape}=\sqrt{2}\ v_{orbital}$

It means that if the speed of a satellite orbiting close to the earth is made $\sqrt{2}$ times (or increased by 41%) then it will escape from the gravitational field.

(vi) If the gravitational force of attraction of the sun on the planet varies as $F\propto\frac{1}{r^n}$ then the orbital velocity varies as $v\propto\frac{1}{\sqrt{r^{n-1}}}$.

Shape of the orbit of a satellite :

(i) If $v<v_0$, then the satellite does not remain in its circular path rather it traces a spiral path and ultimately falls on earth.

(ii) If $v=v_0$ then the satellite revolves in a circular path/orbit.

(iii) If $v>v_0$ but $<v_e\,(v_0<v<v_e)$ then the satellite will revolve round the earth in elliptical orbit.

(iv) If $v=v_e$ then the satellite will move along a parabolic path and escape out of the gravitational field of earth.

(v) If $v>v_e$ then the satellite will move along a hyperbolic path and escape out of the gravitational field of earth.

Time Period of Satellite

It is the time taken by satellite to go once around the earth.

$$\therefore\ T=\frac{\text{Circumference of the orbit}}{\text{orbital velocity}}$$

$$\Rightarrow T=\frac{2\pi r}{v}=2\pi r\sqrt{\frac{r}{GM}} \qquad \left[\text{As } v=\sqrt{\frac{GM}{r}}\right]$$

$$\Rightarrow T=2\pi\sqrt{\frac{r^3}{GM}}=2\pi\sqrt{\frac{r^3}{gR^2}} \quad [\text{As } GM=gR^2]$$

$$\Rightarrow T=2\pi\sqrt{\frac{(R+h)^3}{gR^2}}=2\pi\sqrt{\frac{R}{g}}\left(1+\frac{h}{R}\right)^{3/2} \quad [\text{As } r=R+h]$$

(i) From $T=2\pi\sqrt{\frac{r^3}{GM}}$, it is clear that time period is independent of the mass of orbiting body and depends on the mass of central body and radius of the orbit

(ii) $T=2\pi\sqrt{\frac{r^3}{GM}} \Rightarrow T^2=\frac{4\pi^2}{GM}r^3$ i.e., $T^2\propto r^3$

This is in accordance with Kepler's third law of planetary motion r becomes a (semi major axis) if the orbit is elliptic.

(iii) Time period of nearby satellite,

From $T=2\pi\sqrt{\frac{r^3}{GM}}=2\pi\sqrt{\frac{R^3}{gR^2}}=2\pi\sqrt{\frac{R}{g}}$

[As $h=0$ and $GM=gR^2$]

For earth $R=6400km$ and $g=9.8m/s^2$

$T=84.6$ minute $\approx 1.4\ hr$

(iv) Time period of nearby satellite in terms of density of planet can be given as

$$T=2\pi\sqrt{\frac{r^3}{GM}}=2\pi\sqrt{\frac{R^3}{GM}}=\frac{2\pi(R^3)^{1/2}}{\left[G.\frac{4}{3}\pi R^3\rho\right]^{1/2}}=\sqrt{\frac{3\pi}{G\rho}}$$

(v) If the gravitational force of attraction of the sun on the planet varies as $F\propto\frac{1}{r^n}$ then the time period varies as $T\propto r^{\frac{n+1}{2}}$

(vi) If there is a satellite in the equatorial plane rotating in the direction of earth's rotation from west to east, then for an observer, on the earth, angular velocity of satellite will be $(\omega_S-\omega_E)$. The time interval between the two consecutive appearances overhead will be

$$T=\frac{2\pi}{\omega_s-\omega_E}=\frac{T_ST_E}{T_E-T_S} \qquad \left[\text{As } T=\frac{2\pi}{\omega}\right]$$

If $\omega_S=\omega_E$, $T=\infty$ i.e. satellite will appear stationary relative to earth. Such satellites are called geostationary satellites.

Height of Satellite

As we know, time period of satellite $T=2\pi\sqrt{\frac{r^3}{GM}}=2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$

By squaring and rearranging both sides $\frac{gR^2T^2}{4\pi^2}=(R+h)^3$

$$\Rightarrow h=\left(\frac{T^2gR^2}{4\pi^2}\right)^{1/3}-R$$

By knowing the value of time period we can calculate the height of satellite from the surface of the earth.

Geostationary Satellite

The satellite which appears stationary relative to earth is called geostationary or geosynchronous satellite, communication satellite.

A geostationary satellite always stays over the same place above the earth such a satellite is never at rest. Such a satellite appears stationary due to its zero relative velocity w.r.t. that place on earth.

The orbit of a geostationary satellite is known as the parking orbit.

(i) It should revolve in an orbit concentric and coplanar with the equatorial plane.

(ii) Its sense of rotation should be same as that of earth about its own axis *i.e.*, in anti-clockwise direction (from west to east).

(iii) Its period of revolution around the earth should be same as that of earth about its own axis.

$\therefore\ T = 24\ hr = 86400\ sec$

(iv) Height of geostationary satellite

As $T = 2\pi\sqrt{\frac{r^3}{GM}} \Rightarrow 2\pi\sqrt{\frac{(R+h)^3}{GM}} = 24hr$

Substituting the value of G and M we get $R + h = r = 42000\ km = 7R$

$\therefore$ height of geostationary satellite from the surface of earh $h = 6R = 36000\ km$

(v) Orbital velocity of geo stationary satellite can be calculated by $v = \sqrt{\frac{GM}{r}}$

Substituting the value of G and M we get $v = 3.08\ km/sec$

Angular Momentum of Satellite

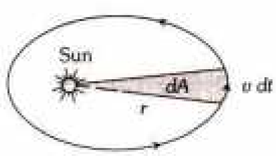
Angular momentum of satellite $L = mvr$

$$\Rightarrow L = m\sqrt{\frac{GM}{r}}\, r \quad [\text{As } v = \sqrt{\frac{GM}{r}}\ ]$$

$$\therefore\quad L = \sqrt{m^2GMr}$$

i.e., Angular momentum of satellite depends on both the mass of orbiting and central body as well as the radius of orbit.

(i) In case of satellite motion, force is central so torque = 0 and hence angular momentum of satellite is conserved *i.e.*, L = constant



(ii) In case of satellite motion as aerial velocity

$$\frac{dA}{dt} = \frac{1}{2}\frac{(r)(vdt)}{dt} = \frac{1}{2}rv$$

$$\Rightarrow \frac{dA}{dt} = \frac{L}{2m} \qquad [\text{As } L = mvr\ ]$$

But as L = constant, $\therefore$ areal velocity (dA/dt) = constant which is Kepler's II law

i.e., Kepler's II law or constancy of areal velocity is a consequence of conservation of angular momentum.

Energy of Satellite

When a satellite revolves around a planet in its orbit, it possesses both potential energy (due to its position against gravitational pull of earth) and kinetic energy (due to orbital motion).

(1) Potential energy : $U = mV = \frac{-GMm}{r} = \frac{-L^2}{mr^2}$

$$\left[\text{As } V = \frac{-GM}{r}, L^2 = m^2GMr\right]$$

(2) Kinetic energy : $K = \frac{1}{2}mv^2 = \frac{GMm}{2r} = \frac{L^2}{2mr^2}$

$$\left[\text{As } v = \sqrt{\frac{GM}{r}}\right]$$

(3) Total energy :

$$E = U + K = \frac{-GMm}{r} + \frac{GMm}{2r} = \frac{-GMm}{2r} = \frac{-L^2}{2mr^2}$$

(i) Kinetic energy, potential energy or total energy of a satellite depends on the mass of the satellite and the central body and also on the radius of the orbit.

(ii) From the above expressions we can say that

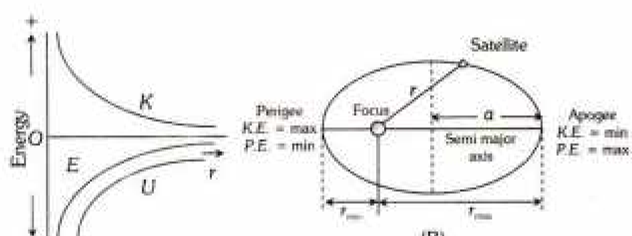
Kinetic energy (K) = – (Total energy)

Potential energy (U) = 2 (Total energy)

Potential energy (U) = – 2 (Kinetic energy)

(iii) Energy graph for a satellite

(iv) Energy distribution in elliptical orbit



(A) (B)

(v) If the orbit of a satellite is elliptic then

(a) Total energy $(E) = \frac{-GMm}{2a}$ = constant ; where a is semi-major axis .

(b) Kinetic energy (K) will be maximum when the satellite is closest to the central body (at perigee) and minimum when it is farthest from the central body (at apogee)

(c) Potential energy (U) will be minimum when kinetic energy = maximum i.e., the satellite is closest to the central body (at perigee) and maximum when kinetic energy = minimum i.e., the satellite is farthest from the central body (at apogee).

(vi) Binding Energy : Total energy of a satellite in its orbit is negative. Negative energy means that the satellite is bound to the central body by an attractive force and energy must be supplied to remove it from the orbit to infinity. The energy required to remove the satellite from its orbit to infinity is called Binding Energy of the system, *i.e.*,

$$\text{Binding Energy (B.E.)} = -E = \frac{GMm}{2r}$$

Change in the Orbit of Satellite

When the satellite is transferred to a higher orbit $(r_2 > r_1)$ then variation in different quantities can be shown by the following table

Quantities	Variation	Relation with r
Orbital velocity	Decreases	$v \propto \frac{1}{\sqrt{r}}$
Time period	Increases	$T \propto r^{3/2}$
Linear momentum	Decreases	$P \propto \frac{1}{\sqrt{r}}$
Angular momentum	Increases	$L \propto \sqrt{r}$
Kinetic energy	Decreases	$K \propto \frac{1}{r}$
Potential energy	Increases	$U \propto -\frac{1}{r}$
Total energy	Increases	$E \propto -\frac{1}{r}$
Binding energy	Decreases	$BE \propto \frac{1}{r}$

Note: ❑ Work done in changing the orbit $W = E_2 - E_1$

$$W = \left(-\frac{GMm}{2r_2}\right) - \left(-\frac{GMm}{2r_1}\right)$$

$$W = \frac{GMm}{2}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

Weightlessness

The weight of a body is the force with which it is attracted towards the centre of earth. When a body is stationary with respect to the earth, its weight equals the gravity. This weight of the body is known as its static or true weight.

We become conscious of our weight, only when our weight (which is gravity) is opposed by some other object. Actually, the secret of measuring the weight of a body with a weighing machine lies in the fact that as we place the body on the machine, the weighing machine opposes the weight of the body. The reaction of the weighing machine to the body gives the measure of the weight of the body.

The state of weightlessness can be observed in the following situations.

(1) **When objects fall freely under gravity** : For example, a lift falling freely, or an airship showing a feat in which it falls freely for a few seconds during its flight, are in state of weightlessness.

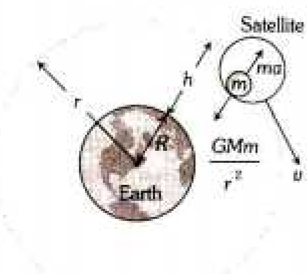
(2) **When a satellite revolves in its orbit around the earth** : Weightlessness poses many serious problems to the astronauts. It becomes quite difficult for them to control their movements. Everything in the satellite has to be kept tied down. Creation of artificial gravity is the answer to this problem.

(3) **When bodies are at null points in outer space** : On a body projected up, the pull of the earth goes on decreasing, but at the same time the gravitational pull of the moon on the body goes on increasing. At one particular position, the two gravitational pulls may be equal and opposite and the net pull on the body becomes zero. This is zero gravity region or the null point and the body in question is said to appear weightless.

Weightlessness in a Satellite

A satellite, which does not produce its own gravity moves around the earth in a circular orbit under the action of gravity. The acceleration of satellite is $\frac{GM}{r^2}$ towards the centre of earth.

If a body of mass m placed on a surface inside a satellite moving around the earth. Then force on the body are



(i) The gravitational pull of earth $= \frac{GMm}{r^2}$

(ii) The reaction by the surface $= R$

By Newton's law $\frac{GmM}{r^2} - R = m\,a$

$$\frac{GmM}{r^2} - R = m\left(\frac{GM}{r^2}\right)$$

$\therefore R = 0$

Thus the surface does not exert any force on the body and hence its apparent weight is zero.

A body needs no support to stay at rest in the satellite and hence all position are equally comfortable. Such a state is called weightlessness.

Examples under condition of weightlessness :

(i) One will find it difficult to control his movement, without weight he will tend to float freely. To get from one spot to the other he will have to push himself away from the walls or some other fixed objects.

(ii) As everything is in free fall, so objects are at rest relative to each other, i.e., if a table is withdrawn from below an object, the object will remain where it was without any support.

(iii) If a glass of water is tilted and glass is pulled out, the liquid in the shape of container will float and will not flow because of surface tension.

(iv) If one tries to strike a match, the head will light but the stick will not burn. This is because in this situation convection currents will not be set up which supply oxygen for combustion

(v) If one tries to perform simple pendulum experiment, the pendulum will not oscillate. It is because there will not be any restoring torque and so $T = 2\pi\sqrt{(L/g')} = \infty$. [As $g' = 0$]

(vi) Condition of weightlessness can be experienced only when the mass of satellite is negligible so that it does not produce its own gravity.

e.g. Moon is a satellite of earth but due to its own weight it applies gravitational force of attraction on the body placed on its surface and hence weight of the body will not be equal to zero at the surface of the moon.

Tips & Tricks

- Gravity holds the atmosphere around to the earth.
- If the earth were at one fourth the present distance from the sun, the duration of the year will be one eighth of the present year.
- If a packet is just released from an artificial satellite, it does not fall to the earth. On the other hand it will continue orbiting along with the satellite.
- Astronauts orbiting around the earth cannot use a pendulum clock. however, they can use spring clock
- To the astronauts in space, the sky appears black due to the absence of atmosphere above them.
- The gravitational force is much smaller than the electrical force because the value of G is very very small.
- The dimensional formula of gravitational field is same as that of acceleration due to gravity.
- A body in gravitational field has maximum binding energy when it is at rest.
- The moon is the natural satellite of the earth, but a man does not feel weightlessness on the surface of the moon. This is because, the mass of the moon is very large and it exerts a gravitational force on the man. On the other hand, the mass of the artificial satellite is very small and it exerts negligible or no gravitational force on the astronaut, so astronaut feels weightlessness in the artificial satellite but not on the moon.
- All other planets except mercury and pluto revolve around the sun in almost circular orbits.
- If the radius of planet decreases by $x\%$ keeping the mass constant. The acceleration due to gravity on its surface increases by $2x\%$.
- If the mass of a planet increases by $x\%$ keeping radius constant, the acceleration due to gravity on its surface increases by $x\%$.
- If the density of the planet decreases by $x\%$, keeping the radius constant, the acceleration due to gravity decreases by $x\%$.
- If the radius of the planet decreases by $x\%$, keeping the density constant, the acceleration due to gravity decreases by $x\%$.
- For the planets orbiting around the sun, angular speed, linear speed, kinetic energy etc. change with time but angular momentum remains constant.
- The ratio of inertial mass to gravitational mass is 1.
- Inertial mass m becomes infinite if the body moves with velocity of light.
- Intensity of gravitational field inside a shell is zero.
- If two spheres of same material, mass and radius are put in contact, the gravitational attraction between them is directly proportional to the fourth power of their radius.

There is no atmosphere on the moon because escape velocity on the moon is less than the *rms* velocity of the gas molecules.

- Two satellites are orbiting in circular orbits of radii r_1 and r_2. Their orbital speeds are in the ratio : $\upsilon_1/\upsilon_2 = (r_2/r_1)^{1/2}$. It is independent to their masses
- An object will experience weightlessness at equator, if the angular speed of the earth about its axis becomes more than (1/800) rad s^{-1}.
- Orbital velocity vary near the surface of the earth and is about 7.92 kms^{-1}.
- Greater the height of the satellite, smaller is the orbital velocity.
- Orbital velocity is independent of the mass of the satellite.
- If the altitude of the satellite is n times the radius of the earth, then the orbital velocity will be $(1/\sqrt{1+n})$ times the orbital velocity near the surface of the earth.
- If the radius of the orbit of a sattelite is n times the radius of the earth, then its orbital velocity will be $(1/\sqrt{n})$ the orbital velocity near the surface of the earth.
- The centripetal acceleration of the satellite is equal to the acceleration due to gravity.
- When velocity of the satellite increases, its kinetic energy increases and hence total energy becomes less negative. That is the satellite begins to revolve in orbit of greater radius.