S H M

Q1. The displacement-time equation of a particle executing SHM is $x = r\sin(\omega t + \phi)$. At time $t = 0$, position of the particle is $x = r/2$ and it is moving along negative x-direction. What is the value of ϕ ?

Ans. At $t = 0, x = r/2$;

so $\frac{r}{2} = r\sin\phi$ or $\sin\phi = \frac{1}{2}$

or $\phi = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

Velocity, $v = \frac{dx}{dt} = r\omega\cos(\omega t + \phi)$

At $t = 0, v = r\omega\cos\phi$

Here, $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ and $\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$

As, v is negative at $t = 0$, ϕ must be $\frac{5\pi}{6}$ rad.

Q2. A particle is executing S.H.M. given by

$$x = 5\cos\left(\frac{2\pi t}{T} + \phi\right)$$

The period of vibration is 20 s. At $t = 0$, the particle is displaced +2 units. Determine

(i) its initial phase and

(ii) the phase angle corresponding to displacement of +3 units.

Ans. Given, $x = 5\cos\left(\frac{2\pi t}{T} + \phi\right)$;

(i) When $t = 0, x = +2$ units,

$\therefore \quad 2 = 5\cos\left(\frac{2\pi\times 0}{T} + \phi\right)$

or $\cos\phi = \frac{2}{5} = 0.4 = \cos 66.4°$

or $\phi = 66.4°$

(ii) $3 = 5\cos\left(\frac{2\pi t}{T} + \phi\right)$

or $\cos\left(\frac{2\pi t}{T} + \phi\right) = \frac{3}{5} = 0.6 = \cos 51.3°$

$\therefore$ Phase angle $= \left(\frac{2\pi t}{T} + \phi\right) = 51.3°$

Q3. A body oscillates with S.H.M., according to the equation

$$x = (5.0 \text{ m}) \cos[(2\pi \text{ rad s}^{-1})t + \pi/4]$$

At $t = 1.5$ s, calculate the (a) displacement (b) speed and (c) acceleration of the body.

Ans. $\omega = 2\pi$ rad/s; $T = 1$ s; $t = 1.5$ s

(a) displacement,

$$\begin{aligned} x &= (5.0)\cos[2\pi\times 1.5 + \pi/4] \\ &= 5.0\cos(3\pi + \pi/4) \\ &= -5.0\cos\pi/4 \\ &= -5.0\times 0.707 \\ &= -3.535 \text{ m} \end{aligned}$$

(b) velocity, $v = \frac{dx}{dt}$

$$\begin{aligned} &= -5.0\times 2\pi\sin(2\pi t + \pi/4) \\ &= -5.0\times 2\pi\sin(2\pi\times 1.5 + \pi/4) \\ &= 5.0\times 2\pi\times\sin\pi/4 \\ &= 5.0\times 2\times\frac{22}{7}\times 0.707 \\ &= 22.22 \text{ ms}^{-1}. \end{aligned}$$

(c) Acceleration $A = \frac{dv}{dt}$

$$\begin{aligned} &= -5.0\times 4\pi^2\cos\left(2\pi t + \frac{\pi}{4}\right) \\ &= -5.0\times 4\times\left(\frac{22}{7}\right)^2\cos\left[2\pi\times 1.5 + \frac{\pi}{4}\right] \\ &= 5.0\times 4\times\left(\frac{22}{7}\right)^2\times 0.707 \\ &= 139.56 \text{ ms}^{-2}. \end{aligned}$$

Q4. Show that when a particle is moving in S.H.M., its velocity at a distance $\sqrt{3}/2$ of its amplitude from the central position is half its velocity in central position.

Ans. Here, $y = \frac{\sqrt{3}}{2}a$

$$v = \omega\sqrt{a^2 - y^2} = \omega\sqrt{a^2 - \frac{3a^2}{4}} = \frac{\omega a}{2} = \frac{v_m}{2}$$

Q5. A particle is moving in S.H.M. in a straight line. When the distance of the particle from equilibrium position has values x_1 and x_2, the corresponding values of velocities are u_1 and u_2. Show that time period of vibration is

$$T = 2\pi\left[\frac{x_2^2 - x_1^2}{u_1^2 - u_2^2}\right]^{1/2}$$

Ans. As $v^2 = \omega^2 (a^2 - y^2)$;

so $\quad u_1^2 = \omega^2 a^2 - \omega^2 x_1^2 \quad$ ...(i)

and $\quad u_2^2 = \omega^2 a^2 - \omega^2 x_2 \quad$ ...(ii)

Subtracting (ii) from (i),

$$u_1^2 - u_2^2 = \omega^2 (x_2^2 - x_1^2)$$

or $$u_1^2 - u_2^2 = \frac{4\pi^2}{T^2}(x_2^2 - x_1^2)$$

or $$T = 2\pi\left[\frac{x_2^2 - x_1^2}{u_1^2 - u_2^2}\right]^{1/2}$$

Q6. A particle moving with S.H.M. in a straight line has a speed of 6 m/s when 4 m from the centre of oscillation and a speed of 8 m/s when 3 m from the centre of oscillation. Find the amplitude of oscillation and the shortest time taken by the particle in moving from the extreme position to a point mid way between the extreme position and the centre.

Ans. $v^2 = \omega^2 (r^2 - y^2)$

(i) $\quad 6^2 = \omega^2 (r^2 - 4^2)$

(ii) $\quad 8^2 = \omega^2 (r^2 - 3^2)$

or $$\frac{64}{36} = \frac{r^2 - 9}{r^2 - 16}$$

On solving, $r = \pm 5$ m and $\omega = 2\ s^{-1}$

For the given displacement, $x = \dfrac{r}{2}$

As $\quad x = r\cos\omega t.$

$\therefore \quad \dfrac{r}{2} = r\cos 2t$

or $\quad \cos 2t = \dfrac{1}{2} = \cos\dfrac{\pi}{3}$

or $\quad t = \pi/6$ s

Q7. Maximum velocity of a particle in SHM is 10 cm/s. What is the average velocity during motion from one extreme position to other extreme position?

Ans. Max. velocity, $v_m = r\,\omega = r 2\pi/T \quad$ ...(i)

$$\text{Avg. vel.} = \frac{\text{total displacement}}{\text{total time}}$$

$$\therefore \quad v_{av} = \frac{2r}{T/2} = \frac{4r}{T} = 4\left(\frac{v_m}{2\pi}\right)$$

[from (i)]

$$= \frac{2v_m}{\pi} = \frac{2\times 10}{\pi} = \frac{20}{\pi}\ \text{cm/s.}$$

Q8. Two particles execute SHM of the same amplitude and frequency along close parallel lines. They pass each other moving in opposite directions, each time their displacement is half their amplitude. What is their phase difference?

Ans. In SHM, $x = r\sin(\omega t + \phi)$

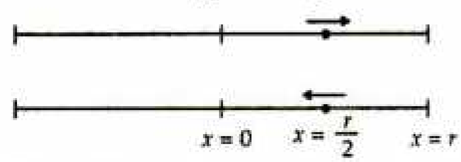
velocity, $v = \dfrac{dx}{dt} = r\,\omega\cos(\omega t + \phi)$

At $t = 0$, $x = r/2$; then

$$\frac{r}{2} = r\sin\phi$$

or $$\sin\phi = \frac{1}{2} = \sin\frac{\pi}{6} \text{ or } \sin\frac{5\pi}{6}$$

$$\therefore \quad \phi = \frac{\pi}{6} \quad \text{or} \quad \frac{5\pi}{6}.$$



If $\phi = \pi/6$, displacement and velocity both are +ve at $t = 0$. When $\phi = 5\pi/6$, displacement is +ve and velocity is –ve. Therefore, displacement-time equations of two particles will be

$$x_1 = r\sin(\omega t + \pi/6)$$
$$x_2 = r\sin(\omega t + 5\pi/6)$$

$\therefore$ Phase difference

$$d\phi = \frac{5\pi}{6} - \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}\ \text{rad.}$$

Q9. A particle executes S.H.M. of period 8 s. After what time of its passing through the mean position will be energy be half kinetic and half potential?

Ans. Given; P.E. = K.E. *i.e.*

$$\frac{1}{2} m \omega^2 y^2 = \frac{1}{2} m \omega^2 (a^2 - y^2)$$

or $\quad y^2 = a^2 - y^2$

i.e. $\quad y = a/\sqrt{2}$

Now $y = a \sin \omega t = a \sin (2\pi/T)\, t$.

So $\quad a/\sqrt{2} = a \sin 2\pi t/8$

or $\quad \sin\frac{\pi t}{4} = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4}$

or $\quad \frac{\pi t}{4} = \frac{\pi}{4} \quad$ or $\quad t = 1$ s.

Q10. A pendulum clock normally shows correct time. On an extremely cold day, its length decreases by 0.2%. Compute the error in time per day.

Ans. The correct time period of pendulum clock is 2 seconds. Let L be its correct length.

$$\therefore \quad 2 = 2\pi\sqrt{\frac{L}{g}} \qquad \ldots\text{(i)}$$

Decrease in length $= 0.2\% = \frac{0.2}{100}L$

Length after contraction,

$$l = L - \frac{0.2}{100}L = L\left(1 - \frac{0.2}{100}\right)$$

New time period t will be,

$$t = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{L}{g}\left(1 - \frac{0.2}{100}\right)}$$

Diving (ii) by (i),

$$\frac{t}{2} = \left(1 - \frac{0.2}{100}\right)^{1/2}$$

or $\quad t = 2\left(1 - \frac{0.2}{100}\right)^{1/2}$

$$= 2\left(1 - \frac{1}{2}\times\frac{0.2}{100} + \ldots\right)$$

or $\quad t = \left(2 - \frac{0.2}{100}\right)$s.

which is less than 2 seconds.

Therefore, the clock gains time. Time gained in 2 seconds $= \frac{0.2}{100}$ s.

Total time gained in 1 day $(= 24 \times 60 \times 60\text{ s})$

$$= \frac{0.2}{100} \times \frac{24\times60\times60}{2} = 86.4\text{ s.}$$

Q11. A spring compressed by 0.1 m develops a restoring force 10 N. A body of mass 4 kg is placed on it. Deduce (i) the force constant of the spring (ii) the depression of the spring under the weight of the body (take $g = 10$ N/kg) and (iii) the period of oscillation, if the body is disturbed.

Ans. Here, $F = 10$ N; $\Delta l = 0.1$ m; $m = 4$ kg.

(i) $\quad k = \frac{F}{\Delta l} = \frac{10}{0.1} = 100\text{ N m}^{-1}$.

(ii) $\quad y = \frac{mg}{k} = \frac{4\times10}{100} = 0.4$ m

(iii) $\quad T = 2\pi\sqrt{\frac{m}{k}} = 2 \times \frac{22}{7}\sqrt{\frac{4}{100}} = 1.26$ s.

Q12. A body of mass 12 kg is suspended by a coil spring of natural length 50 cm and force constant $2.0 \times 10^3\text{ Nm}^{-1}$. What is the stretched length of the spring? If the body is pulled down further stretching the spring to a length of 5.9 cm and then released, what is the frequency of oscillation of the suspended mass?

(Neglect the mass of the spring).

Ans. $m = 12$ kg; original length $l = 50$ cm;

$k = 2.0 \times 10^3\text{ Nm}^{-1}$.

As $\quad F = ky$

$$\therefore \quad y = \frac{F}{k} = \frac{mg}{k} = \frac{12\times9.8}{2\times10^3}$$

$$= 5.9 \times 10^{-2}\text{ m} = 5.9\text{ cm}$$

$\therefore$ Stretched length of the spring $= l + y$

$$= 50 + 5.9 = 55.9\text{ cm}$$

Frequency of oscillations, $\nu = \frac{1}{T}$

$$= \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\times3.14}\sqrt{\frac{2\times10^3}{12}}$$

$= 2.06\text{ s}^{-1}$

Q13. Infinite springs with force constants k, $2k$, $4k$ and $8k$.... respectively are connected in series. What will be the effective force constant of the springs, when $k = 10$ N/m.

Ans. When springs are in series, the effective force constant k is given by

$$\frac{1}{K} = \frac{1}{k} + \frac{1}{2k} + \frac{1}{4k} + \frac{1}{8k} + \ldots\ldots$$

$$= \frac{1}{k}\left(1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots\right)$$

$$= \frac{1}{k}\left(\frac{1}{1-\frac{1}{2}}\right) = \frac{2}{k}$$

or $K = \frac{k}{2} = \frac{10}{2} = 5$ N/m.

Q14. A spring has spring constant $k = 15$ N/cm. It is cut into 3 equal parts, which are joined in parallel. What is the spring constant of the combination?

Ans. Let k be the spring constant of each of the three parts into which the spring is cut. As initially these three parts of spring are connected in series, so

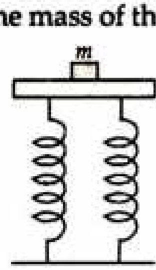
$$\frac{1}{K} = \frac{1}{k} + \frac{1}{k} + \frac{1}{k} = \frac{3}{k}$$

or $k = 3K$

When these three parts of a spring are connected in parallel, the effective spring constant K′ is given by

$K' = k + k + k = 3k = 9K = 9 \times 15 = 135$ N/cm.

Q15. A tray of mass 12 kg is supported by two identical springs as shown. When the tray is pressed down slightly and released, it executes S.H.M. with a time period of 1.5 s. What is the force constant of each spring? When a block of mass m is placed on the tray, the period of S.H.M. changes to 3 s. What is the mass of the block?



Ans. Let k_1 be the force constant of each spring. The total force constant of the system,

$K = k_1 + k_1 = 2k_1$

$$T_1 = 2\pi\sqrt{\frac{m'}{K}},$$

$$\therefore K = \frac{4\pi^2 m'}{T_1^2} = \frac{4 \times (22/7)^2 \times 12}{(1.5)^2} = 211 \text{Nm}^{-1}$$

$$\therefore k_1 = \frac{K}{2} = \frac{211}{2} = 105.5 \text{ Nm}^{-1}$$

Total mass, $M = m + m' = m + 12$

$$T_2 = 2\pi\sqrt{\frac{m+m'}{K}}$$

or $$K = \frac{4\pi^2(m+m')}{T_2^2}$$

$$\therefore \frac{4\pi^2 m'}{T_1^2} = \frac{4\pi^2(m+m')}{T_2^2}$$

or $$m = \frac{T_2^2}{T_1^2}m' - m'$$

$$= \frac{3^2}{(1.5)^2} \times 12 - 12 = 36 \text{ kg}$$

Q16. Two point masses of 3 kg and 1 kg are attached to opposite ends of a horizontal spring whose spring constant is 300 Nm^{-1}. Find the natural frequency of vibration of the system.

Ans. The reduced mass of the system,

$$m = \frac{m_1 \times m_2}{m_1 + m_2} = \frac{3 \times 1}{3+1} = \frac{3}{4} \text{ kg.}$$

So inertia factor $= m = \frac{3}{4}$ kg.

Spring factor, $k = 300$ Nm^{-1}.

$$\text{Frequency, } \nu = \frac{1}{2\pi}\sqrt{\frac{\text{spring factor}}{\text{inertia factor}}}$$

$$= \frac{1}{2 \times (22/7)}\sqrt{\frac{300}{(3/4)}}$$

$= 3.2$ Hz.

Q17. The vertical motion of a huge piston in a machine is simple harmonic with a frequency of 0.50 s^{-1}. A block of 10 kg is placed on the piston. What is the maximum amplitude of the piston's S.H.M. for the block and the piston to remain together?

Ans. As $\quad \nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$\therefore \quad k = 4\pi^2 m \nu^2$

For maximum displacement, $y_{max} = a$.

Maximum restoring force, $F = -ka = -mg$.

or $\quad a = \frac{mg}{k} = \frac{mg}{4\pi^2 m\nu^2} = \frac{g}{4\pi^2\nu^2}$

$$= \frac{9.8}{4\times(3.14)^2\times(0.50)^2} = 0.99 \text{ m.}$$

Q18. A simple pendulum with a brass bob has a time period T. The bob is now immersed in a non viscous liquid and oscillated. If the density of the liquid is 1/9 that of brass, find the time period of the same pendulum.

Ans. Let V be the volume and ρ be the density of the brass bob.

Mass of the bob $m = V\rho$

and weight of bob $= V\rho g$.

Buoyancy force of liquid on bob

$$= V(\rho/9)g = V\rho g/9$$

The effective weight of bob in liquid

$$= V\rho g - V\rho g/9 = 8V\rho g/9$$

$\therefore$ Acceleration,

$$g' = \frac{8V\rho g/9}{m} = \frac{8V\rho g/9}{V\rho} = \frac{8g}{9}$$

Time period of the bob

$$= 2\pi\sqrt{\frac{l}{g'}} = 2\pi\sqrt{\frac{l}{(8g/9)}}$$

$$= 2\pi\sqrt{\frac{l}{g}}\times\frac{3}{\sqrt{8}} = \frac{3T}{\sqrt{8}}$$

Q19. A man stands on a weighing machine placed on a horizontal platform. The machine reads 50 kg. By means of a suitable mechanism, the platform is made to executive harmonic vibrations up and down with a frequency of 2 vibrations per second. What will be the effect on the reading of the weighing machine? The amplitude of vibration of platform is 5 cm. Take $g = 10$ ms^{-1}.

Ans. Here, $m = 50$ kg, $\nu = 2$ s^{-1}, $a = 5$ cm $= 0.05$ m

Max. acceleration, $a_{max} = \omega^2 a$

$$= (2\pi\nu)^2 a = 4\pi^2\nu^2 a$$

$$= 4\times\left(\frac{22}{7}\right)^2\times(2)^2\times 0.05$$

$$= 7.9 \text{ ms}^{-2}$$

$\therefore$ Max. force on the man

$$= m(g + a_{max}) = 50\,(10 + 7.9)$$

$$= 895.0 \text{ N} = 89.5 \text{ kgf.}$$

Min. force on the man

$$= m(g - a_{max})$$

$$= 50\,(10 - 7.9) = 105.0 = 10.5 \text{ kgf.}$$

Hence the reading of the weighing machine varies between 10.5 kgf and 89.5 kgf.

Q20. *On an average a human heart is found to beat 75 times in a minute. Calculate its frequency and period.*

Sol. The beat frequency of heart = 75/(1 min)

= 75/(60 s)

= 1.25 s^{-1}

= 1.25 Hz

The time period T = 1/(1.25 s^{-1})

= 0.8 s

Q21. *Which of the following functions of time represent (a) periodic and (b) non-periodic motion? Give the period for each case of periodic motion [ω is any positive constant].*

(i) $\sin \omega t + \cos \omega t$

(ii) $\sin \omega t + \cos 2\omega t + \sin 4\omega t$

(iii) $e^{-\omega t}$

(iv) $\log(\omega t)$

Sol. (i) $\sin \omega t + \cos \omega t$ is a periodic function, it can also be written as $\sqrt{2}\sin(\omega t + \pi/4)$.

$\sqrt{2}\sin(\omega t + \pi/4) = \sqrt{2}\sin(\omega t + \pi/4 + 2\pi)$

$= \sqrt{2}\sin[\omega(t + 2\pi/\omega) + \pi/4]$

The periodic time of the function is $2\pi/\omega$.

(ii) This is an example of a periodic motion. Each term represents a periodic function with a different angular frequency.

$\sin \omega t$ has a period $T_0 = 2\pi/\omega$;

$\cos 2\omega t$ has a period $\pi/\omega = T_0/2$; and

$\sin 4\omega t$ has a period $2\pi/4\omega = T_0/4$.

The smallest interval of time after which the sum of the three terms repeats is T_0, and thus the sum is a periodic function with a period $2\pi/\omega$.

(iii) The function $e^{-\omega t}$ is not periodic, it decreases monotonically with increasing time.

(iv) The function $\log(\omega t)$ increases monotonically with time t. It, therefore, is a non-periodic function.

Q22. *Which of the following functions of time represent (a) simple harmonic motion and (b) periodic but not simple harmonic? Give the period for each case.*

(a) $\sin \omega t - \cos \omega t$ (b) $\sin^2 \omega t$

Sol. (a) $\sin \omega t - \cos \omega t$

$= \sin \omega t - \sin(\pi/2 - \omega t)$

$= 2\cos(\pi/4)\sin(\omega t - \pi/4)$

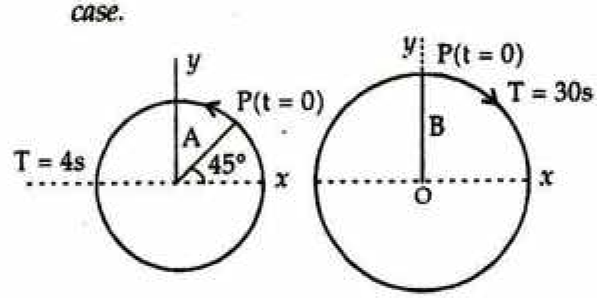
$= \sqrt{2}\sin(\omega t - \pi/4)$

This function represents a simple harmonic motion having a period $T = 2\pi/\omega$ and a phase angle $(-\pi/4)$ or $(7\pi/4)$

(b) $\sin^2 \omega t = ½ - ½\cos 2\omega t$

The function is periodic having a period $T = \pi/\omega$. It also represents a harmonic motion with the point of equilibrium occurring at ½ instead of zero.

Q23. *Figure given below depicts two circular motions. The radius of the circle, the period of revolution, the initial position and the sense of revolution are indicated on the figures. Obtain the simple harmonic motions of the x-projection of the radius vector of the rotating particle P in each case.*



Sol. (a) At $t = 0$, OP makes an angle of $45° = \pi/4$ rad with x-axis. After time t, it covers an angle $\frac{2\pi}{T}t$ in the anticlockwise sense, and makes an angle of $\frac{2\pi}{T}t + \frac{\pi}{4}$ with the x-axis.

The projection of OP on the x-axis at time t is given by,

$x(t) = A\cos\left(\frac{2\pi}{T}t + \frac{\pi}{4}\right)$

For T = 4 s,

$x(t) = A\cos\left(\frac{2\pi}{T}t + \frac{\pi}{4}\right)$

which is a SHM of amplitude A, period 4 s, and an initial phase $= \frac{\pi}{4}$.

(b) In this case, at $t = 0$, OP makes an angle of 90° $= \frac{\pi}{2}$ with the x-axis. After a time t, it covers an angle of $\frac{2\pi}{T}t$ in the clockwise sense and makes an angle of $\left(\frac{\pi}{2} - \frac{2\pi}{T}t\right)$ with the x-axis. The projection of OP on the x-axis at time t is given by

$$x(t) = B\cos\left(\frac{\pi}{2} - \frac{2\pi}{T}t\right)$$

$$= B\sin\left(\frac{2\pi}{T}t\right)$$

For $T = 30$ s,

$$x(t) = B\sin\left(\frac{\pi}{15}t\right)$$

$$x(t) = B\cos\left(\frac{\pi}{15}t - \frac{\pi}{2}\right)$$

This represents a SHM of amplitude B, period 30 s, and an initial phase of $-\frac{\pi}{2}$.

Q24. *A body oscillates with SHM according to the equation (in SI units),*

$$x = (5)\cos[2\pi t + \pi/4].$$

At $t = 1.5$ s, calculate the (a) displacement, (b) speed and (c) acceleration of the body.

Sol. The angular frequency ω of the body $= 2\pi\ s^{-1}$ and its time period $T = 1$ s. At $t = 1.5$ s,

(a) displacement

$$= (5.0\ m)\cos[(2\pi\ s^{-1}) \times 1.5\ s + \pi/4]$$
$$= (5.0\ m)\cos[(3\pi + \pi/4)]$$
$$= -5.0 \times 0.707\ m$$
$$= -3.535\ m$$

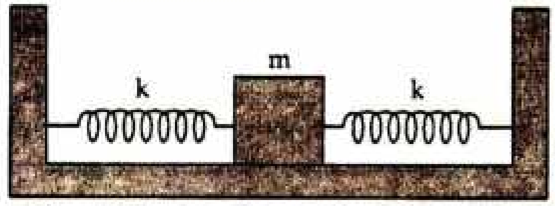
(b) $v(t) = -\omega A\sin(\omega t + \phi)$

$$= -(5.0\ m)(2\pi\ s^{-1})\sin[(2\pi\ s^{-1}) \times 1.5\ s + \pi/4]$$
$$= -(5.0\ m)(2\pi\ s^{-1})\sin[(3\pi + \pi/4)]$$
$$= 10\pi \times 0.707\ m\ s^{-1}$$
$$= 22\ m\ s^{-1}$$

(c) $a(t) = -\omega^2 x(t)$

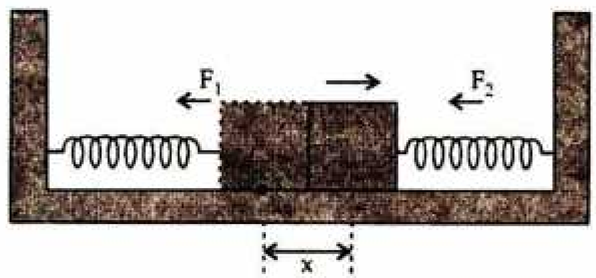
$$= -(2\pi\ s^{-1})^2 \times \text{displacement}$$
$$= -(2\pi\ s^{-1})^2 \times (-3.535\ m)$$
$$= 140\ m\ s^{-2}$$

Q25. *Two identical springs of spring constant k are attached to a block of mass m and to fixed supports as shown.*



Show that when the mass is displaced from its equilibrium position on either side, it executes a simple harmonic motion. Find the period of oscillations.

Sol. Let the mass be displaced by a small distance x to the right side of the equilibrium position, as shown.



The spring on the left side gets elongated by a length equal to x and that on the right side gets compressed by the same length. The forces acting on the mass are then,

$F_1 = -kx$ (force exerted by the spring on the left side, trying to pull the mass towards the mean position)

$F_2 = -kx$ (force exerted by the spring on the right side, trying to push the mass towards the mean position)

The net force, F, acting on the mass is,

$F = -2kx$

Thus the force acting on the mass is proportional to the displacement and is directed towards the mean position;

therefore, the motion executed by the mass is simple harmonic. The time period of oscillations is,

$$T = 2\pi\sqrt{\frac{m}{2k}}$$

Q26. *A block whose mass is 1 kg is fastened to a spring. The spring has a spring constant of 50 N m^{-1}. The block is pulled to a distance x = 10 cm from its equilibrium position at x = 0 on a frictionless surface from rest at t = 0. Calculate the kinetic, potential and total energies of the block when it is 5 cm away from the mean position.*

Sol. The block executes SHM, its angular frequency is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{50\,\text{N m}^{-1}}{1\,\text{kg}}} = 7.07\ \text{rad s}^{-1}$$

Its displacement at any time t is then given by,

$x(t) = 0.1 \cos(7.07t)$

Therefore, when the particle is 5 cm away from the mean position,

$0.05 = 0.1 \cos(7.07t)$

Or $\cos(7.07t) = 0.5$ and hence

$$\sin(7.07t) = \frac{\sqrt{3}}{2} = 0.866$$

Then the velocity of the block at $x = 5$ cm is

$= 0.1 \times 7.07 \times 0.866\ \text{m s}^{-1}$

$= 0.61\ \text{m s}^{-1}$

Hence the K.E. of the block,

$= \frac{1}{2}mv^2 = \frac{1}{2}[1\text{kg} \times (0.6123\ \text{m s}^{-1})^2] = 0.19$ J

The P.E. of the block,

$= \frac{1}{2}kx^2 = \frac{1}{2}(50\ \text{N m}^{-1} \times 0.05\ \text{m} \times 0.05\ \text{m})$

$= 0.0625$ J

The total energy of the block at $x = 5$ cm,

= K.E. + P.E. = 0.25 J

At maximum displacement, K.E. is zero and hence the total energy of the system is equal to the P.E. Therefore, the total energy of the system,

$= \frac{1}{2}(50\ \text{N m}^{-1} \times 0.1\ \text{m} \times 0.1\ \text{m})$

$= 0.25$ J

Q27. *A 5 kg collar is attached to a spring of spring constant 500 N m^{-1}. It slides without friction over a horizontal rod. The collar is displaced from its equilibrium position by 10.0 cm and released. Calculate*

(a) the period of oscillation,

(b) the maximum speed and

(c) maximum acceleration of the collar.

Sol. (a) $T = 2\pi\sqrt{\frac{m}{k}}$

$$= 2\pi\sqrt{\frac{5.0\ \text{kg}}{500\ \text{N m}^{-1}}}$$

$= (2\pi/10)$ s

$= 0.63$ s

(b) $v(t) = -A\omega \sin(\omega t + \phi)$

$v_{max} = A\omega$

$$= 0.1 \times \sqrt{\frac{k}{m}}$$

$$= 0.1 \times \sqrt{\frac{500\ \text{N m}^{-1}}{5\ \text{kg}}}$$

$= 1\ \text{m s}^{-1}$

and it occurs at $x = 0$

(c) $a(t) = -\omega^2 x(t)$

$$= -\frac{k}{m}x(t)$$

Therefore the maximum acceleration is,

$a_{max} = \omega^2 A$

$$= \frac{500\ \text{N m}^{-1}}{5\ \text{kg}} \times 0.1\ \text{m} = 10\ \text{m s}^{-2}$$

Q28. *What is the length of a simple pendulum, which ticks seconds ?*

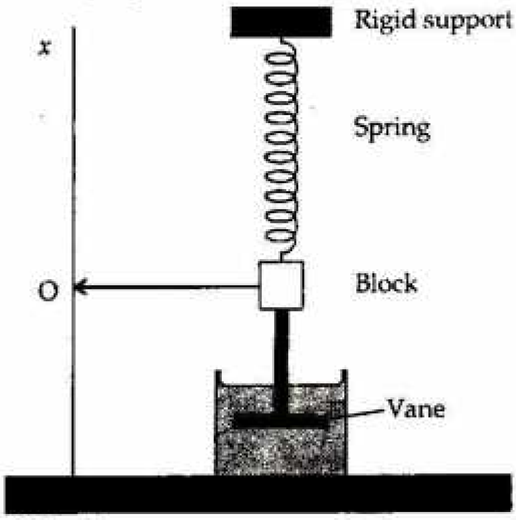
Sol. $T = 2\pi\sqrt{\frac{L}{g}}$

$$L = \frac{gT^2}{4\pi^2}$$

The time period of a simple pendulum, which ticks seconds, is 2 s.

$$L = \frac{9.8\ \text{m s}^{-2} \times 4\text{s}^2}{4\pi^2} = 1\text{m}$$

Q29. *For the damped oscillator shown, the mass m of the block is 200 g, k = 90 N m^{-1} and the damping constant b is 40 g s^{-1}.*



Calculate

(a) the period of oscillation,

(b) time taken for its amplitude of vibrations to drop to half of its initial value and

(c) the time taken for its mechanical energy to drop to half its initial value.

Sol. (a) $km = 90\times0.2 = 18\ \text{kgNm}^{-1} = 18\ \text{kg}^2\,\text{s}^{-2}$

$\sqrt{km} = 4.243\ \text{kg s}^{-1}$, and $b = 0.04\ \text{kg s}^{-1}$.

Therefore b is much less than $\sqrt{km}$. Hence the time period T is given by

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2\pi\sqrt{\frac{0.2\ \text{kg}}{90\ \text{N m}^{-1}}}$$

$$= 0.3\ \text{s}$$

(b) $$T_{1/2} = \frac{\ln(1/2)}{b/2m} = \frac{0.693}{40}\times2\times200s = 6.93\ \text{s}$$

(c) $$E(t) = \frac{1}{2}kA^2e^{-bt/m}$$

$E(t_{1/2})/E(0) = \exp(-bt_{1/2}/m)$

Or ½ $= \exp(-bt_{1/2}/m)$

$\ln(1/2) = -(bt_{1/2}/m)$

Or $$t_{1/2} = \frac{0.693}{40\ \text{g s}^{-1}}\times200\ \text{g} = 3.46\ \text{s}$$

Q30. *A particle is in linear simple harmonic motion between two points A and B, 10 cm apart. Take the direction from A to B as the positive direction and give the signs of velocity, acceleration and force on the particle when it is*

(i) at the end A,

(ii) at the end B,

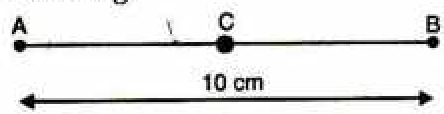
(iii) at the mid-point of AB going towards A,

(iv) at 2 cm away from B going towards A,

(v) at 3 cm away from A going towards B, and

(vi) at 6 cm away from B going towards A.

Sol. See Fig..



(*i*) At the end *A*, velocity is zero ; both acceleration and force are +ve, *i.e.*, towards *AB*.

(*ii*) At the end *B*, velocity is zero ; both acceleration and force are negative, *i.e.*, towards *BA*.

(*iii*) At the midpoint of *AB*, *i.e.*, at C going towards *A* velocity is negative whereas both acceleration and force are zero.

(*iv*) At 2 cm away from *B* going towards *A*, velocity, acceleration and force are all negative.

(*v*) At 3 cm away from *A* going towards *B*, all the quantities are positive.

(*vi*) At 6 cm away from *B* going towards *A*, velocity is negative ; both the acceleration and the force are positive.

Q31. *Which of the following relationships between the acceleration a and the displacement x of a particle involve simple harmonic motion.*

(i) $a = 0.7x$

(ii) $a = -200x^2$

(iii) $a = -10x$

(iv) $a = 100x^3$.

Sol. (*i*) $a = 0.7x$

Comparing it with relation of acceleration with the displacement in SHM

$$a = -\omega^2x$$

We find that the motion is not SHM.

(ii) $a = -200\ x^2$

This motion is also not a SHM for the same reason.

(iii) $a = -10x$

This is a SHM.

(iv) $a = 100\ x^3$

This equation does not represent a SHM.

Q32. *The motion of a particle executing simple harmonic motion is described by the displacement function*

$$x(t) = A\cos(\omega t + \phi).$$

If the initial (t = 0) position of the particle is 1 cm and its initial velocity is ω cm/s, what are its amplitude and initial phase angle ? The angular frequency of the particle is π s^{-1}. If instead of the cosine function, we choose the sine function to describe the SHM : x = B sin (ωt + α), what are the amplitude and initial phase of the particle with the above initial conditions.

Sol. Given $x(t) = A\cos(\omega t + \phi)$ ...(i)

At $t = 0, x(0) = 1$ cm

$$\left(\frac{dx}{dt}\right)_{\text{at } t=0} = \omega \text{ cm/s.}$$

$$A = ?$$

From (i) and given values at $t = 0$

We have,

$$1 = A\cos(\omega \times 0 + \phi)$$

$$\Rightarrow \quad A\cos\phi = 1 \quad ...(ii)$$

Differentiating equation (i), we get

$$v = \frac{dx}{dt} = -A\omega\sin(\omega t + \phi) \quad ...(iii)$$

Putting the initial condition value in (iii)

i.e. $\omega = -A\omega\sin(\omega \times 0 + \phi)$

$$\Rightarrow \quad A\sin\phi = -1 \quad ...(iv)$$

Squaring equation (ii) and (iv) and adding,

$$A^2\cos^2\phi + A^2\sin^2\phi = (1)^2 + (-1)^2$$

$$\Rightarrow \quad A^2 = 2$$

$$\Rightarrow \quad A = \pm\sqrt{2}$$

$$\Rightarrow \quad \boxed{A = +\sqrt{2}} \quad \text{[Neglecting –ve value]}$$

Dividing equation (iv) by (ii)

$$\frac{A\sin\phi}{A\cos\phi} = \frac{-1}{1} \Rightarrow \tan\phi = -1 \Rightarrow \boxed{\phi = \frac{3\pi}{4} \text{ or } \frac{7\pi}{4}}$$

For $x = B\sin(\omega t + \alpha)$ ...(A)

at $t = 0, x = 1$

$$\Rightarrow \quad 1 = B\sin(\omega \times 0 + \alpha)$$

$$\Rightarrow \quad 1 = B\sin\alpha \quad ...(a)$$

Differentiating (A) w.r.t t

$$v = \frac{dx}{dt} = B\omega\cos(\omega t + \alpha)$$

Applying initial conditions

i.e. at $t = 0, v = \omega$ cm/sec.

$$\omega = B\omega\cos(\omega \times 0 + \alpha)$$

$$\Rightarrow \quad 1 = B\cos\alpha \quad ...(b)$$

Squaring (a) and (b) and adding, we get

$$\Rightarrow \quad B^2\cos^2\alpha + B^2\sin^2\alpha = 1^2 + 1^2$$

$$\Rightarrow \quad B^2 = 2$$

$$\Rightarrow \quad B = \pm\sqrt{2}$$

$$\Rightarrow \quad \boxed{B = +\sqrt{2}} \quad \text{[Neglecting –ve value]}$$

Dividing equation (a) by (b)

$$\frac{1}{1} = \frac{B\sin\alpha}{B\cos\alpha}$$

$$\Rightarrow \quad \boxed{\alpha = \frac{\pi}{4} \text{ or } \frac{5\pi}{4}}$$

Q33. *A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this balance, when displaced and released, oscillates with a period of 0.60 s. What is the weight of the body?*

Sol. We know that the time period of oscillation of a loaded spring is given by

$$T = 2\pi\sqrt{\frac{m}{k}}$$

or

$$m = \frac{T^2 k}{4\pi^2}. \quad ...(i)$$

When a mass of 50 kg is put on the spring, it extends to 20 cm (equal to the length of the scale), i.e.,

$$50 \times g = k \times 20 \text{ cm} = k \times 0.2$$

$$\therefore \quad k = \frac{50 \times 9.8}{0.2} \text{ N/m.}$$

and $T = 0.60$ s (given)

Substituting the values in equation (i), we get

$$m = \frac{(0.6)^2 \times 50 \times 9.8}{0.2 \times 4 \times (3.142)^2} = 22.34 \text{ kg.}$$

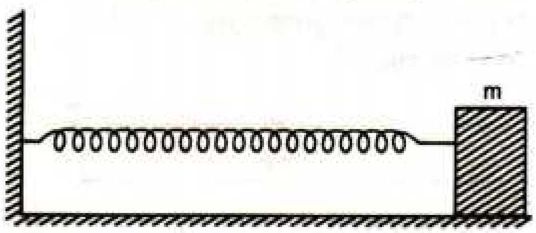
$\therefore$ *Weight of the body* w = **22.34 kgf** **Ans.**

Q34. *A spring of force constant 1200 N/m is mounted on a horizontal table as shown (Figure). A mass of 3.0 kg is attached to the free end of the spring. The mass is then pulled sideways to a distance of 2.0 cm and released.*

(*i*) *What is the frequency of oscillation of the mass?*

(*ii*) *What is the maximum acceleration of the mass?*

(*iii*) *What is the maximum speed of the mass?*



Sol. (*i*) The frequency of oscillation is given by

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Here, $k = 1200$ N/m and $m = 3.0$ kg.

$$\therefore \quad \nu = \frac{1}{2\pi}\sqrt{\frac{1200}{3.0}} = \frac{1}{2\pi}\sqrt{400}$$

$$= \frac{20}{2\pi} = 3.18 = 3.2/s.$$

The frequency of oscillation of the mass is **3.2/s. Ans.**

(*ii*) *The maximum acceleration*

$$|a|_{max} = \frac{k}{m}|x|_{max} = \frac{k}{m}A.$$

Here, A, the amplitude of the oscillation is 2.0 cm $= 2.0 \times 10^{-2}$ m.

$$\therefore \quad |a|_{max} = \frac{1200}{3.0} \times 2.0 \times 10^{-2}$$

$= \mathbf{8.0\ m/s^2}$ **Ans.**

(*iii*) Since displacement is given by

$$x(t) = A\cos(\omega t + \phi),$$

Velocity, $$v = \frac{dx}{dt} = -A\omega \sin(\omega t + \phi)$$

$$= -A\sqrt{\frac{k}{m}}\sin(\omega t + \phi)$$

$$\therefore \quad |v|_{max} = A\sqrt{\frac{k}{m}}$$

[$\because$ for max value, $\sin(\omega t + \phi) = 1$]

$$= 2.0 \times 10^{-2} \times \sqrt{\frac{1200}{3.0}}$$

$= 0.40$ m/s

$\therefore$ *The maximum velocity is* **0.40 m/s. Ans.**

Q35. *In* Q34. *what is*

(*a*) *the speed of the mass when the spring is compressed by 1.0 cm,*

(*b*) *potential energy of the mass when it momentarily comes to rest,*

(*c*) *total energy of the oscillating mass ?*

Sol. (*a*) The speed of the mass is given by

$$v = \omega\sqrt{A^2 - x^2}$$

Here, $A = 2$ cm $= 0.02$ m, $x = 1.0$ cm $= 0.01$ m

and $$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1200}{3.0}} = 20/s.$$

$\therefore$ *The speed of the mass when the spring is compressed by* 1.0 cm $= 20 \times \sqrt{(0.02)^2 - (0.01)^2}$

$= 20 \times \sqrt{4-1} \times 10^{-2} = 0.20 \times \sqrt{3}$

= 0.35 m/s. Ans.

(*b*) The mass comes momentarily at rest at the extreme position, *i.e.* at $x = 0.02$ m.

$\therefore$ *The potential energy* $= \frac{1}{2} \times k \times x^2$

$= \frac{1}{2} \times 1200 \times (0.02)^2$

= 0.24 J. Ans.

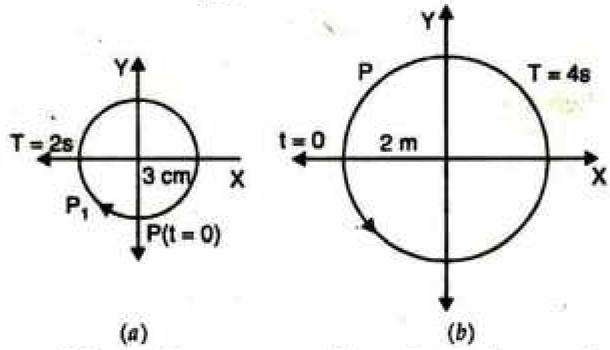
(c) The total energy $= \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

At the extreme position $v = 0$ and $x = 0.02$.

$\therefore$ *The total energy* $= \frac{1}{2} \times 1200 \times (0.02)^2$

= 0.24 J Ans.

Q36. *Figures (Figure) below correspond to two circular motions. The radius of the circle, the period of revolution, the initial position, and the sense of revolutions (i.e. clockwise or anticlockwise) are indicated on each figure.*



Obtain the corresponding simple harmonic motions of the x-projection of the radius vector of the revolving particle P, in each case.

Sol. (*a*) When we drop a perpendicular on the X-axis of the circle from the position of the particle marked $t = 0$, we get $x = 0$ and, therefore, the initial phase $\phi = 0$. Further, as *t increases*, the particle moves to a position towards left say to P_1. Now, when we drop the perpendicular on to the X-axis, we get negative value of x. Also, since the radius of the circle is 3 cm, the amplitude of the SHM is 3 cm and $\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi/s$.

Thus, equation of simple harmonic motion is

$x = A \sin \omega t$

$= -3 \sin \pi t$. (where x is in cm). **Ans.**

(*b*) Here $A = 2$ m, $T = 4s$

or $\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \frac{\pi/2}{s}$

and $\phi = 3\pi/2$.

$\therefore$ $x = A \sin(\omega t + \phi)$

$$= 2 \sin\left(\frac{\pi}{2}t + \frac{3\pi}{2}\right) = -2\cos\frac{\pi}{2}t,$$

(where x is in m) **Ans.**

Q37. *Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial (t = 0) position of the particle, the radius of the circle and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anticlockwise in every case : (x is in cm and t is in s).*

(*i*) $x = -2 \sin\left(3t + \frac{\pi}{3}\right)$ (*ii*) $x = \cos\left(\frac{\pi}{6} - t\right)$

(*iii*) $x = 3 \sin\left(2\pi t + \frac{\pi}{4}\right)$ (*iv*) $x = 2 \cos \pi t$.

Sol. If we express each function in the form

$$x = A\cos(\omega t + \phi) \quad ...(i)$$

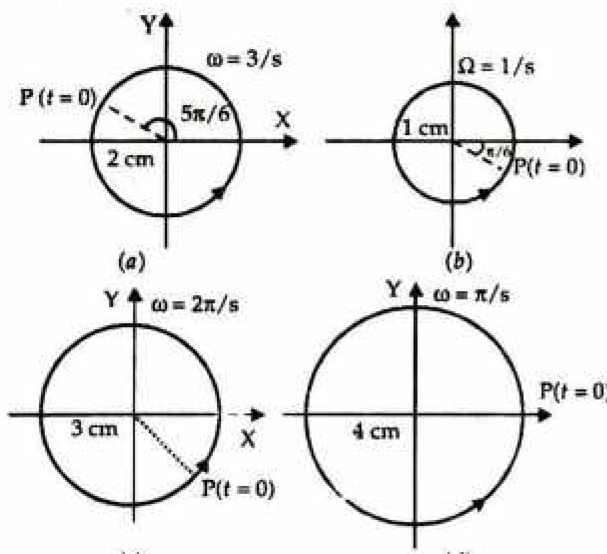
then ϕ represents the angle which the initial radius vector of the particle makes with the positive direction of x-axis.

(*a*) Comparing the general equation (*i*) with the given equation $x = -2 \sin(3t + \pi/3)$ $= 2\cos\left(\frac{\pi}{2} + 3t + \frac{\pi}{3}\right)$, we note that $A = 2$, $\omega = 3$ and $\phi = \frac{\pi}{2} + \frac{\pi}{3}$.

Hence, the reference circle will be as shown in Fig. (*a*)

(*b*) In this case $x = \cos\left(\frac{\pi}{6} - t\right) = \cos\left(t - \frac{\pi}{6}\right)$
$[\because \cos\theta = \cos(-\theta)]$

Comparing it with equation (*i*), we get $A = 1$, $\omega = 1$ and $\phi = -\pi/6$. The circle is shown in Fig. (*b*).



(*c*) Here $x = 3 \sin\left(2\pi t + \frac{\pi}{4}\right)$

$$= 3 \cos\left(2\pi t + \frac{3\pi}{2} + \frac{\pi}{4}\right)$$

Comparing it with equation (*i*), we get

$A = 3$, $\omega = 2\pi$ and $\phi = \frac{3\pi}{2} + \frac{\pi}{4}$. The circle is given by Fig. (*c*) (*d*) Here $x = 2\cos \pi t$.

Comparing it with equation (*i*), we have

$A = 2, \omega = \pi$ and $\phi = 0$.

The circle is depicted in Fig. (*d*).

Q38. Figure (a) *shows a spring of force constant k clamped rigidly at one end and a mass M attached to its free end. The spring is stretched by a force F at its free end. Figure (b) shows the same spring with both ends free and attached to a mass m at either end. Each end of the spring in Figure (b) is stretched by the same force F.*

(*i*) *What is the maximum extension of the spring in the two cases ?*

(*ii*) *If the mass in Fig. (a) and the two masses in Fig. (b) are released free, what is the period of oscillation in each case?*

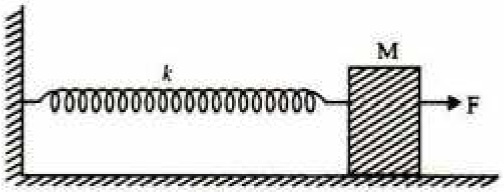


Fig. (a)

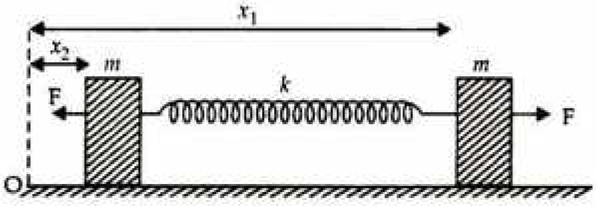


Fig. (b)

Sol. (*i*) The maximum extension in both the cases will be F/k, where k is spring constant.

(*ii*) Let x_1 and x_2 be the positions of the two masses from some arbitrary point O [see Figure (*b*)]. The force exerted by the spring on the two masses are F and $-F$. These two forces are equal ($=kx$) and opposite.

Applying Newton's law to the masses, we get

$$m\frac{d^2x_1}{dt^2} = -kx \quad ...(i) \quad \text{and} \quad m\frac{d^2x_2}{dt^2} = kx. \quad ...(ii)$$

Subtracting equation (*ii*) from equation (*i*), we get

$$m\left(\frac{d^2x_1}{dt^2} - \frac{d^2x_2}{dt^2}\right) = -2kx$$

or $$m\frac{d^2}{dt^2}(x_1 - x_2) = -2kx \quad ...(iii)$$

The change in length of the spring

$$x = (x_1 - x_2) - l \quad ...(iv)$$

Since l is constant, from equation (*iv*), we have

$$\frac{d^2x}{dt^2} = \frac{d^2}{dt^2}(x_1 - x_2)$$

Substituting in equation (*iii*), we get

$$m\frac{d^2x}{dt^2} + 2kx = 0$$

or $$\frac{d^2x}{dt^2} + \frac{2k}{m}x = 0$$

$\therefore$ $$\omega = \sqrt{\frac{2k}{m}}$$

or $$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{2k}}. \quad ...(v)$$

However, in the case (*a*), T will be given by

$$T = 2\pi\sqrt{\frac{m}{k}}. \quad ...(vi)$$

$\therefore$ The periods of oscillation for cases (*a*) and (*b*) are given by equations (*vi*) and (*v*) respectively.

Q39. *The piston in the cylinder head of a locomotive has a stroke (twice the amplitude) of 1.0 m. If the piston moves with simple harmonic motion with an angular frequency of 200 rev/min, what is its maximum speed ?*

Sol. Given A = ½ m; ω = 200 rev/min.

$$v_{max} = A\omega = \tfrac{1}{2} \times 200$$

$$\boxed{v_{max} = 100 \text{ m/min}}$$

Q40. *The acceleration due to gravity on the surface of the moon is 1.7 m/s². What is the time-period of a simple pendulum on the moon if its time-period on the earth is 3.5 s? (g on the surface of earth = 9.8 m/s²)*

Sol. If T and T' are time-periods of a pendulum of length l on the earth and the moon respectively, we have

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \text{and} \quad T' = 2\pi\sqrt{\frac{l}{g'}}$$

where g and g' are the values of acceleration due to gravity on the earth and the moon respectively.

$$\therefore \quad \frac{T'}{T} = \sqrt{\frac{g}{g'}} \text{ or } T' = T\sqrt{\frac{g}{g'}}.$$

Now, $g = 9.8$ m/s², $g' = 1.7$ m/s² and $T = 3.5$ s.

Substituting the values, we get

$$T' = 3.5\sqrt{\frac{9.8}{1.7}} = 8.430 = 8.4 \text{ s}$$

$\therefore$ *The time period of the pendulum on the moon is* **8.4 s. Ans.**

Q41. *A simple pendulum of length l and having a bob of mass M is suspended in a car. The car is moving on a circular track of radius R with a uniform speed v. If the pendulum makes small oscillations in a radial direction about its equilibrium position, what will be its time period.*

Sol. Centripetal acceleration,

$$a_c = \frac{v^2}{R} \quad \text{[acting horizontally]}$$

and acceleration due to gravity = g which is acting vertically downwards.

Effective acceleration due to gravity.

$$g_0 = \sqrt{g^2 + \frac{v^4}{R^2}}$$

$\therefore$ Time period, $T = 2\pi\sqrt{\frac{l}{g_0}}$

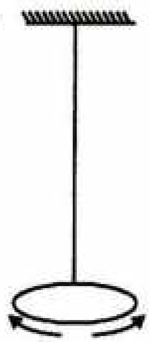
$$\boxed{T = 2\pi\sqrt{\frac{l}{(g^2 + v^4/R^2)}}}$$

Q42. *A circular disc of mass 10 kg is suspended by a wire attached to its centre. The wire is twisted by rotating the disk and released. The period of torsional oscillation is found to be 1.5 s. The radius of the disk is 15 cm. Determine the torsional spring constant of the wire. (Torsional spring constant α is defined by the relation $J = -\alpha\theta$ where J is the restoring couple and θ the angle of twist).*

Sol. The time period of torsional oscillations of a body (suspended by a wire) about its axis of suspension can be shown to be

$$T = 2\pi\sqrt{\frac{I}{\alpha}} \quad \text{or} \quad \alpha = \frac{4\pi^2 I}{T^2} \qquad ...(i)$$

where I is moment of inertia of the body about its axis of oscillation and α is the couple required to twist the wire through unit angle. It is also called spring constant of the wire for torsional oscillations (see Fig.).



Here $T = 1.5$ s,

$$M.I. = I = \frac{1}{2}MR^2 = \frac{1}{2} \times 10 \times (0.15)^2.$$

Substituting the values in equation (i), we get

$$\alpha = \frac{4 \times (3.14)^2 \times 1 \times 10 \times (0.15)^2}{2 \times (1.5)^2}$$

= 1.972 N m/rad.

Q43. *A body describes simple harmonic motion with an amplitude of 5 cm and a period of 0.2 s. Find the acceleration and velocity of the body when the displacement is (a) 5 cm, (b) 3 cm, (c) 0 cm.*

Sol. Here, $a = 5 \text{ cm} = 0.05 \text{ m}$

$$T = 0.2 \text{ s}; \quad \omega = \frac{2\pi}{T} = \frac{2\pi}{0.2} = 10\pi \text{ rad/s}$$

When displacement is y, then

acceleration, $f = -\omega^2 y$

velocity, $V = \omega\sqrt{a^2 - y^2}$

Case (a) When $y = 5 \text{ cm} = 0.05 \text{ m}$

$$f = -(10\pi)^2 \times 0.05 = -5\pi^2 \text{ m/s}^2$$

$$V = 10\pi\sqrt{(0.05)^2 - (0.05)^2} = 0$$

Case (b) When $y = 3 \text{ cm} = 0.03 \text{ m}$

$$f = -(10\pi)^2 \times 0.03 = -3\pi^2 \text{ m/s}^2$$

$$V = 10\pi\sqrt{(0.05)^2 - (0.03)^2}$$

$$= 10\pi \times 0.04 = 0.4\pi \text{ m/s}$$

Case (c) When $y = 0$, $f = -(10\pi)^2 \times 0 = 0$

$$V = 10\pi\sqrt{(0.05)^2 - 0^2}$$

$$= 10\pi \times 0.05 = 0.5\pi \text{ m/s}$$

Q44. *A mass attached to a spring is free to oscillate, with angular velocity ω, in a horizontal plane without friction or damping. It is pulled to a distance x_0 and pushed towards the centre with a velocity v_0 at time t = 0. Determine the amplitude of the resulting oscillations in terms of the parameters ω, x_0 and v_0. [Hint : Start with the equation $x = a\cos(\omega t + \theta)$ and note that the initial velocity is negative.]*

Sol. $x = A\cos(\omega t + \theta)$, $\frac{dx}{dt} = -A\omega\sin(\omega t + \theta)$

When $t = 0$, $x = x_0$ and $\frac{dx}{dt} = -v_0$

$$\therefore \quad x_0 = A\cos\theta \qquad ...(i)$$

$$-v_0 = -A\omega\sin\theta$$

$$\text{or} \quad v_0/\omega = A\sin\theta \qquad ...(ii)$$

Squaring and adding (i) and (ii), we get

$$A = [v_0^2/\omega^2 + x_0^2]^{1/2}$$