SURFACE TENSION

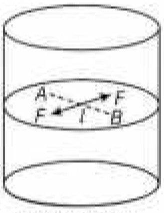
Surface Tension and Surface Energy

Surface tension arises due to the fact that the free surface of a liquid at rest has some additional potential energy.

Surface Tension

Surface tension is the property of liquid at rest by virtue of which a liquid surface tends to occupy a minimum surface area and behaves like stretched membrane.

e.g., A steel needle may be made to float on water though the steel is more dense than water. This is because the water surface acts as a stretched elastic membrane and supports the needle. This property of a liquid is called **surface tension.**



Definition of surface tension

Consider a line *AB* on the free surface of a liquid. The small elements of the surface on this line are in equilibrium because they are acted upon by equal and opposite forces, acting perpendicular to the line from either side as shown in figure.

The force acting on this line is proportional to the length of this line. If l is the length of imaginary line and F the total force on either side of the line, then

$$F \propto l \Rightarrow F = Sl$$

$$S = \frac{F}{l} \quad \text{or} \quad \boxed{\text{Surface tension, } S = \frac{\text{Force}}{\text{Length}}}$$

From this expression, surface tension can be defined as the force acting per unit length of an imaginary line drawn on the liquid surface, the direction of force being perpendicular to this line and tangential to the liquid surface. It is denoted by S and it is a **scalar quantity.**

Units and Dimension of Surface Tension

SI units of surface tension = N/m

CGS unit of surface tension = dyne/cm

Dimension of surface tension

$$= \frac{\text{Force}}{\text{Length}} = \frac{[MLT^{-2}]}{[L]} = [ML^0T^{-2}]$$

Factors Affecting Surface Tension

1. **Temperature** The surface tension of liquid decreases with rise in temperature and *vice-versa. Due to this,*
   (*i*) Hot soup taste better than cold soup as hot soup spread over a large area of tongue.
   (*ii*) Machinery parts get jammed in water as surface tension of lubricating oil increases with decrease in temperature.
2. **Addition of Impurities** The surface tension of liquids changes appreciably with addition of impurities. *e.g.*, Surface tension of water increases with addition of highly soluble substances like NaCl, $ZnSO_4$ etc.

Applications of Surface Tension

(*i*) Rain drops and drops of mercury placed on glass plate are spherical.

(*ii*) Hair of shaving brush/painting brush, when dipped in water spread out, but as soon as it is taken out, its hair stick together.

(*iii*) A greased needle placed gently on the free surface of water in a beaker does not sink.

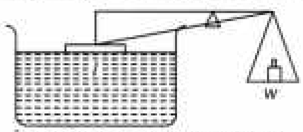
(*iv*) Oil drop spreads on cold water but does not change shape on hot water

Surface Tension of Some Liquids at the Temperatures Indicated with the Heats of the Vapourisation

Liquid	Temperature (in °C)	Surface tension (N/m)	Heat of vaporisation (kJ/mol)
Helium	−270	0.000239	0.115
Oxygen	−183	0.0132	7.1
Ethanol	20	0.0227	40.6
Water	20	0.0727	44.16
Mercury	20	0.4355	63.2

Measuring the Surface Tension of a Liquid

The surface tension of liquid can be measured experimentally as shown in figure. A flat vertical glass plate, below which a vessel filled with some liquid is kept. The plate is balanced by weights on the other side. The vessel is raised slightly until the liquid touches the glass plate and pulls it down because of the force of surface tension. Weights are added till the plate just detected from water.



Measuring surface tension

Suppose the additional weight required is mg.

$$S = (mg/2l)$$

where m is the extra mass and l is the length of the plate edge.

Detergent and Surface Tension

Surface tension has a wide use in daily life. The detergents, used for cleaning the dirty clothes in our home is a very good example of surface tension.

Actually, water cannot wet oil marks on your clothes, that is why water alone cannot remove dirt from your clothes.

The molecule of detergent can attached with water and dirt molecules and they take away the dirt with them when we wash the clothes with detergent.

Surface Energy

The free surface of a liquid always has a tendency to contract and possess minimum surface area. If it is required to increase the surface area of the liquid work has to be done. This **work done** is stored in the surface film of the liquid as its **potential energy**.

This potential energy per unit area of the surface film is called the **surface energy**.

Hence, the surface energy may be defined as the amount of work done in increasing the area of the surface film through unity. Thus,

$$\text{Surface energy} = \frac{\text{Work done in increasing the surface area}}{\text{Increase in surface area}}$$

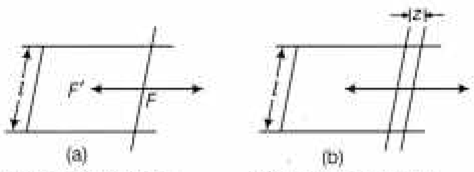
The SI unit of surface energy is J/m^2.

Relation between Surface Energy and Surface Tension

Consider a rectangular frame *PQRS*. Here, wire *QR* is movable. A soap film is formed on the frame. The film pulls the movable wire *QR* inward due to surface tension.

As, $$\text{surface tension} = \frac{\text{Force}}{\text{Length}} = \frac{F'}{2l}$$

$$F' = S \times 2l$$



(a) Equilibrium position of the film (b) Stretched position of the film

If *QR* is moved through a distance z by an external force F very slowly then, some work has to be done against this force.

$\therefore$ External work done = Force × Distance

$$= S \times 2l \times z \qquad [\because F' = F]$$

Increase in surface area of film $= 2l \times z$

[As soap film has two sides]

$$\text{Surface energy} = \frac{\text{Work done}}{\text{Surface area}} = \frac{S2lz}{2lz} = S$$

So, **value of surface energy of liquid is numerically equal to the value of surface tension.**

Angle of Contact

The surface of liquid near the plane of contact with another medium is in curved shape.

The angle between tangent to the liquid surface at the point of contact and solid surface inside the liquid is called as **angle of contact**. It is denoted by θ.

The value of angle of contact depends on the following factors

(*i*) Nature of the solid and liquid in contact.

(*ii*) Cleanliness of the surface in contact.

(*iii*) Medium above the free surface of the liquid.

(*iv*) Temperature of the liquid

e.g., Water form droplets on lotus leaf shown in Fig. (a) while spreads over a clean plastic plate in Fig. (b).

Consider the three interfacial tensions at all the three interfaces such as

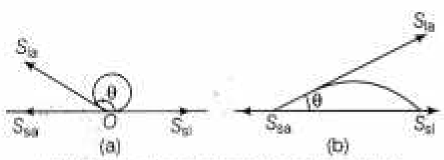
S_{sa} = surface tension between solid and air

S_{la} = surface tension between liquid and air

S_{sl} = surface tension between solid and liquid

At the line of contact, the surface forces between the three media must be in equilibrium. Resolving S_{la} into two rectangular components, we have $S_{la}\cos\theta$ acts along the surface and $S_{la}\sin\theta$ acts along the perpendicular to the solid surface.

As the liquid on the surface of soild is at rest so the molecules of these interfaces are in equilibrium. Thus, the net force on them is zero.



(a) (b)

Different shapes of water drops with interfacial tensions

For the molecule O to be in equilibrium.

$$S_{sl} + S_{la}\cos\theta = S_{sa}$$

$$\boxed{\cos\theta = \frac{S_{sa} - S_{sl}}{S_{la}}}$$

The following cases arises

(*i*) If the surface tension at the solid-liquid S_{sl}, interface is greater than the surface tension at the liquid-air S_{la} interface, *i.e.*, $S_{sl} > S_{la}$, then $\cos\theta$ is negative and $\theta > 90°$ (the angle of contact is **obtuse angle**). The molecules of a liquid are attracted strongly to themselves and weakly to those of solid. It costs a lot of energy to create a liquid-solid surface. The liquid then does not wet the solid.

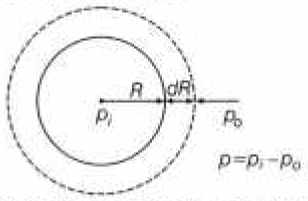
e.g., Water-leaf or glass-mercury interface.

(*ii*) If the surface tension at the solid-liquid S_{sl} interface is less than the surface tension at the liquid-air S_{la} interface, *i.e.*, $S_{sl} < S_{la}$, then $\cos\theta$ is positive and $0 < 90°$ (the angle of contact is **acute angle**). The molecules of the liquid are strongly attracted to those of solid and weakly attracted to themselves. It costless energy to create a liquid-solid surface and liquid wets the solid.

e.g., When soap or detergent is added to water, the angle of contact becomes small.

Excess Pressure Inside a Liquid Drop

Suppose a spherical liquid drop of radius R and S be the surface tension of liquid. Due to its spherical shape, there is an excess pressure p inside the drop over that on outside. This excess pressure acts normally outwards. Due to this pressure, radius increases from R to $R + dR$, then extra surface energy can be determined.



Excess pressure inside a liquid drop

Initial surface area of the liquid $= 4\pi R^2$

Final surface area of the liquid drop

$$= 4\pi(R + dR)^2 = 4\pi(R^2 + 2R\,dR + dR^2)$$

$$= 4\pi R^2 + 8\pi R dR$$

[dR^2 is very small and hence neglected]

Increase in the surface area of liquid drop

$$= 4\pi R^2 + 8\pi R\,dR - 4\pi R^2 = 8\pi R dR$$

External work done in increasing the surface area of the drop

= Increase in surface energy

= Increase in surface area × Surface tension

$= (8\pi R dR) \times S$ ...(i)

But work done

= Excess pressure × Area × Change in radius

$= p \times 4\pi R^2 \times dR$ ...(ii)

From Eqs. (i) and (ii), we get

So, $p \times 4\pi R^2 \times dR = 8\pi R dR S$

Excess pressure, $p = \frac{2S}{R}$

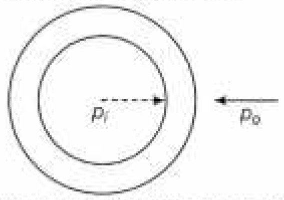
So, **Pressure difference in a drop bubble,** $p_i - p_o = \frac{2S}{R}$

Excess Pressure Inside a Soap Bubble

From the above case,

Increase in surface area $= 8\pi R\,dR$

But a soap bubble has two free surfaces.



Excess pressure inside a soap bubble

So, effective increase in surface area = 2 [Final surface area – initial surface area]

$$= 2[4\pi(R + dR)^2 - 4\pi R^2]$$

$= 2 \times 8\pi R\,dR = 16\pi R\,dR$ ...(i)

External work done in increasing the surface area of the soap bubble = Increase in surface energy

= Increase in surface area × surface tension

$= 16\pi R\,dRS$

But, work done = Force × Change in radius

where Force = Excess pressure × Area

$= (p \times 4\pi r^2)$

So, work done $= p \times 4\pi R^2 \times dR$ ...(ii)

From Eqs. (i) and (ii), we get

$$p \times 4\pi R^2 \times dR = 16\pi R dRS$$

$$p = \frac{4S}{R}$$

Pressure difference inside a soap bubble, $p_i - p_o = \frac{4S}{R}$

Excess pressure inside an air bubble in a liquid is similar to a liquid drop in air, it has only one free spherical surface. Hence, excess pressure is given by

$$p = \frac{2S}{R}$$

- *When an air bubble of radius R lies at a depth h below the free surface of a liquid of density ρ and surface tension S, the excess pressure inside the bubble will be*

$$p = \frac{2S}{R} + h\rho g$$

- *When an ice-skater slides over the surface of smooth ice, some ice melts due to the pressure exerted by the sharp metal edges of the skates. The tiny water droplets act as rigid ball bearings and help the skaters to run along smoothly.*

Worked out Problem

The lower end of a capillary tube of diameter 2 mm is dipped 8 cm below the surface of water in a beaker. What is the pressure required in the tube in order to blow a hemispherical bubble at its end in water? The surface tension of water at temperature of the experiments is $7.30\times10^{-2}\,Nm^{-1}$, 1 atm $= 1.01\times10^{5}$ Pa, density of water $= 1000\ kg/m^3$, $g = 9.80\ ms^{-2}$. Also calculate the excess pressure. [NCERT]

Solution

Step I **Identify the excess pressure in bubble and calculate the outside pressure (p_o) in the bubble**

The excess pressure in a bubble of gas in a liquid is given by $\frac{2s}{r}$.

Given, $h = 0.08$ m, $d = 1000\ kg/m^3$, $g = 9.80\ m/s^2$ and we know that $p_0 = p_a + hdg$

p_0 is outside pressure

$$p_o = 1.01\times10^5\ Pa + 0.08\ m \times 1000\ kg/m^3 \times 9.80\ m/s^2$$

$$p_0 = 1.01784\times10^5\ Pa$$

Step II **Calculate inside pressure required in tube in order to blow a hemispherical bubble**

$$p_i = p_0 + \frac{2s}{r},\ \text{where } s = 7.30\times10^{-2}\ Pa\text{-}m$$

$$p_i = 1.01784\times10^5 + \frac{2\times7.3\times10^{-2}}{10^{-3}}$$

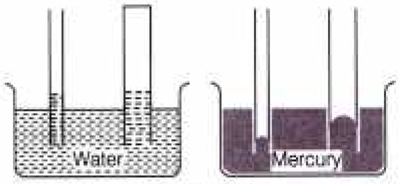
$$= (1.01784 + 0.00146)\times10^5\ Pa$$

$$p_i = 1.02\times10^5\ Pa$$

where, the radius of the bubble is taken to be equal

Capillarity

The term capilla means hair which is Latin word. A tube of very fine (hair-like) bore is called a **capillary tube.**



Capillarity

If a capillary tube of glass is dipped in liquid like water, the liquid rises in the tube, but when the capillary tube is dipped in a liquid like mercury, the level of liquid falls in the tube.

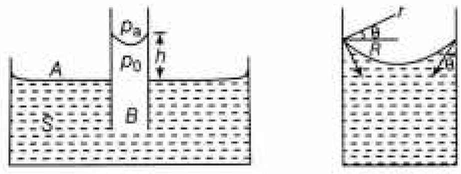
This phenomenon of rise or fall of a liquid in the capillary is called **capillarity.**

Some examples are

1. We use towels for drying our skin.
2. In trees sap rises due to vessels. It is similar to capillary action.

Capillary Rise

One application of the pressure difference across a curved surface is the water rises up in a narrow tube (capillary) inspite of gravity. Consider a capillary of radius R is inserted into a vessel containing water.



(a) (b)

Capillary rise

The surface of water in the capillary becomes concave. It means that there must be a pressure difference between the two sides of the meniscus.

So, $(p_a - p_o) = (2S/r) = 2S/(R\sec\theta) = (2S/R)\cos\theta$ ...(i)

Now, consider two points A and B. According to Pascal's law they must be at the same pressure,

$$p_o + h\rho g = p_A = p_B = p_a$$

So,
$$p_a = p_o + h\rho g$$

$$p_a - p_o = \rho g h \quad \text{...(ii)}$$

(p_a = atmospheric pressure)

From Eqs. (i) and (ii), we get

$$\rho g h = \frac{2S}{R}\cos\theta$$

This is the formula for the rise of liquid in a capillary.

The liquids which met the glass surfaces *e.g.*, water, rises in the capillary and the liquids which do no met the glass surface falls in the capillary.

Some common example of capillarity are

(*i*) Blotting paper absorbs ink due to capillarity.

(*ii*) A towel soaks water on account of capillarity motion.

(*iii*) Oil rises through the wicks due to capillarity.

Example 1 **Water Rise in Capillary**

A capillary of radius 0.05 cm is immersed in water. Find the value of rise of water in capillary if value for the surface tension is 0.073 N/m and angle of contact is 0°.

Solution Given, $S = 0.073\ \text{N/m}$,

$$R = 0.05\ \text{cm} = 5\times10^{-4}\ \text{m}$$

$$\theta = 0^\circ,\ h = ?$$

From the formula, $h = \dfrac{2S\cos\theta}{\rho g R}$

$$= \frac{2\times0.073\times\cos0^\circ}{10^3\times9.8\times5\times10^{-4}} = 0.02979\ \text{m}$$

Worked out **Problem**

In a glass capillary tube water rises upto a height of 10.0 cm while mercury fall down by 5.0 cm in the same capillary. If the angles of contact for mercury glass is 60° and water glass is 0°, then find the ratio of surface tension of mercury and water.

Solution

Step I Write the given quantities for water and mercury.

For water, $h_1 = 10.0\ \text{cm} = 0.1\ \text{m}$

$\rho_1 = 10^3\ \text{kg/m}^3,\ \theta = 0^\circ$

For mercury, $h_2 = 5.0\ \text{cm} = 0.05\ \text{m}$

$\rho_2 = 13.6\times10^3\ \text{kg/m}^3,\ \theta = 60^\circ$

Step II Suppose S_1 and S_2 are the surface tensions for water and mercury respectively then

$$S_1 = \frac{h_1 R\rho_1 g}{2\cos\theta_1} \quad \text{and} \quad S_2 = \frac{h_2 R\rho_2 g}{2\cos\theta_2}$$

Step III Find out the ratio of surface tension of mercury and water.

$$\frac{S_2}{S_1} = \frac{h_2 R\rho_2 g}{2\cos\theta_2}\times\frac{2\cos\theta_1}{h_1 R\rho_1 g}$$

$$\frac{S_2}{S_1} = \frac{h_2\rho_2\cos\theta_1}{h_1\rho_1\cos\theta_2}$$

$$= \frac{0.05\times13.6\times10^3\times\cos0^\circ}{0.1\times1000\times\cos60^\circ} = \frac{0.68}{0.05} = 13.6:1$$

Very Short Answer Type Questions [1 Mark]

1. *If a wet piece of wood burns, then water droplets appear on the other end, why?*

Sol. When a piece of the wood burns, then steam formed and water appear in the form of drops due to surface tension on the other end.

2. *An iceberg weighs 400 tonne. The specific gravity of iceberg is 0.92 and the specific gravity of water is 1.02. What percentage of iceberg is below the water surface?*

Sol. Fraction of the iceberg below the water surface will be

$$\frac{\text{Volume of the immersed part}}{\text{Total volume of the iceberg}}$$

$$= \frac{\text{Density of the substance}}{\text{Density of water}} = \frac{0.92}{1.02} = 0.902 = 90.2\%$$

3. *Why is it difficult to separate the two paper sheets joined with fevicol?*

Sol. Due to the force of adhesion between paper and fevicol.

4. *Find the work done in increasing the radius of a soap bubble from 4 cm to 6 cm. The value of surface tension for the soap solution is 30 dyne cm^{-1}.* [Delhi 11]

Sol. Given, $r_1 = 4\ \text{cm}, r_2 = 6\ \text{cm}, S = 30\ \text{dyne cm}^{-1}$

So, change in surface area $= 2\times4\pi(6^2 - 4^2)$

[Soap solution has two surfaces]

$$= 8\pi\,20 = 160\,\pi\,\text{cm}^2$$

Work done $= S\times$ change in surface area

$$= 30\times160\times3.142$$

$$= 15081.6\ \text{erg}$$

5. *Find the work required to make a soap bubble of radius 0.02 m?*

Sol. Given, $S = 0.03$ N/m

Work done = surface area × surface tension

$$= 2 \times 4\pi r^2 \times S$$
$$= 2 \times 4 \times 3.14 \times (0.02)^2 \times 0.03 = 3 \times 10^{-4}\ \text{J}$$

6. *Why soap bubble bursts after some time?*

Sol. Soap bubble bursts after some time because the pressure inside it become more than the outside pressure.

7. *Why antiseptics have low surface tension value?*

Sol. So that antiseptic could easily spread over the wound and we get proper healing.

Short Answer Type I Questions [2 Marks]

8. *The surface tension and vapour pressure of water at 20°C is 7.28×10^{-2} N/m and 2.33×10^3 Pa, respectively. What is the radius of the smallest spherical water droplet which can form without evaporating at 20° C?* [NCERT]

Sol. Given, surface tension of water $(S) = 7.28 \times 10^{-2}$ N/m

Vapour pressure $(p) = 2.33 \times 10^3$ Pa

The drop will evaporate, if the water pressure is greater than the vapour pressure.

Let a water droplet of radius R can be formed without evaporating. [1]

∴ Vapour pressure = Excess pressure in drop

$$\therefore \quad p = \frac{2S}{R}$$

$$\text{or} \quad R = \frac{2S}{p} = \frac{2 \times 7.28 \times 10^{-2}}{2.33 \times 10^3} \approx 6.25 \times 10^{-5}\ \text{m}$$ [1]

9. *The sap in trees, which consists mainly of water in summer, rises in a system of capillaries of radius $r = 2.5 \times 10^{-5}$m. The surface tension of sap is $S = 7.28 \times 10^{-2}$ N/m and angle of contact is 0°. Does surface tension alone account for the supply of water to the top of all trees?* [NCERT]

Sol. Given, radius $(r) = 2.5 \times 10^{-5}$ m

Surface tension $(S) = 7.28 \times 10^{-2}$ N/m

Angle of contact $(\theta) = 0°$, density of water $(\rho) = 10^3$ kg/m^3

The maximum height to which sap can rise in trees through capillarity action is given by [1]

$$h = \frac{2S \cos\theta}{r\rho g}$$

$$h = \frac{2 \times 7.28 \times 10^{-2} \times \cos 0°}{2.5 \times 10^{-5} \times 1 \times 10^3 \times 9.8} = 0.59\ \text{m}$$

But the height of many trees are more than 0.59 m, therefore, the rise of sap in all trees is not possible through capillarity action alone. [1]

10. *Iceberg floats in water with part of it submerged. What is the fraction of the volume of iceberg submerged, if the density of ice is $\rho_i = 0.917$ gcm^{3}?* [NCERT]

Sol. Given, density of ice $(\rho_i) = 0.917$ g/cm^3

Density of water $(\rho_w) = 1$ g/cm^3

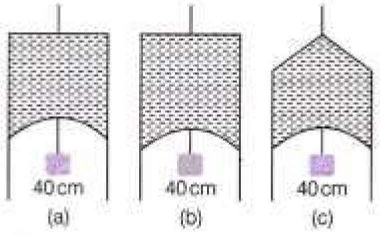
Let V be the total volume of the iceberg and V' of its volume be submerged in water.

In floating condition,

Weight of the iceberg = Weight of the water displaced by the submerged part by ice [1]

$$V\rho_i g = V'\rho_w g \Rightarrow \frac{V'}{V} = \frac{\rho_i}{\rho_w} = \frac{0.917}{1} = 0.917$$ [1]

11. *Fig.(a) shows a thin liquid film supporting a small weight $= 4.5 \times 10^{-2}$N. What is the weight supported by a film of the same liquid at the same temperature in Figs. (b) and (c)? Explain your answer physically.* [NCERT]



Sol. As liquid is same, temperature is same and the length of the film supporting the weight is also same, therefore in Figs. (b) and (c), the film will support same weight *i.e.* 4.5×10^{-2} N. [2]

12. *A liquid drop breaks into 27 small drops. If surface tension of the liquid as S, then find the energy released.*

Sol. Let the radius of larger drop = R

and radius of each small drop = r

Volume of 27 small drops = Volume of the large drop

$$= 27 \times \frac{4}{3} \times \pi r^3 = \frac{4}{3}\pi R^3$$

So, $r = R/3$

Surface area of large drop = $4\pi R^2$ [1]

Surface area of 27 small drops = $27 \times 4\pi r^2$

$$= 27 \times 4\pi \left(\frac{R}{3}\right)^2$$

$$= 12\pi R^2$$

$\therefore$ Increase in surface area = $12\pi R^2 - 4\pi R^2 = 8\pi R^2$

Increase in energy = Increase in surface area × Surface tension

$$= 8\pi R^2 \times S$$ [1]

13. *A liquid drop of radius 4 mm breaks into 1000 identical drops. Find the change in surface energy.* $S = 0.07\ \text{Nm}^{-1}$.

Sol. Volume of 1000 small drops = Volume of large drop

$$1000 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

$$r = \frac{R}{10}$$

Surface area of large drop = $4\pi R^2$ [1]

Surface area of 1000 drop = $4\pi \times 1000\, r^2 = 40\pi R^2$

$\therefore$ Increase in surface area = $(40 - 4)\pi R^2 = 36\,\pi R^2$

The increase in surface energy

= Surface tension × increase in surface area

$= 36\,\pi R^2 \times 0.07 = 36 \times 3.14 \times (4 \times 10^{-3})^2 \times 0.07$

$= 1.26 \times 10^{-4}$ J [1]

14. *Two soap bubbles of radii 6 cm and 8 cm coalesce to form a single bubble. Find the radius of the new bubble?*

Sol. Surface energy of first bubble

= Surface tension × Surface area

$= 2 \times 4\pi R_1^2 S = 8\pi R_1^2 S$ [1]

Surface energy of second bubble = $8\pi R_2^2 S$

Let the radius of the new bubble is R. So, the surface energy of new bubble = $8\pi R^2 S$

By the law of conservation of energy,

$$8\pi R^2 S = 8\pi R_1^2 S + 8\pi R_2^2 S$$

$$R^2 = R_1^2 + R_2^2$$

$$= 36 + 64$$

$\therefore$ $R^2 = 100\ \text{cm}^2$

$\therefore$ $R = 10$ cm [1]

Short Answer Type II Questions [3 Marks]

15. *Two mercury droplets of radii 0.1 cm and 0.2 cm collapse into one single drop. What amount of energy is released? The surface tension of mercury* $S = 435.5 \times 10^{-3}$ N/m.

When two or more droplets collapse to form a bigger drop, then its surface area decreases and energy is released equal to S∆A. [NCERT]

Sol. Radii of mercury droplets $r_1 = 0.1\ \text{cm} = 1 \times 10^{-3}$ m

$r_2 = 0.2\ \text{cm} = 2 \times 10^{-3}$ m

Surface tension $(S) = 435.5 \times 10^{-3}$ N/m

Let the radius of the big drop formed by collapsing be R.

$\therefore$ Volume of big drop = Volume of small droplets [1]

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3$$

or $R^3 = r_1^3 + r_2^3$

$= (0.1)^3 + (0.2)^3$

$= 0.001 + 0.008 = 0.009$

or $R = 0.21\ \text{cm} = 2.1 \times 10^{-3}$ m

∴ Change in surface area

$$\Delta A = 4\pi R^2 - (4\pi r_1^2 + 4\pi r_2^2)$$
$$= 4\pi [R^2 - (r_1^2 + r_2^2)] \quad [1]$$

∴ Energy released = $S \cdot \Delta A$

$$= S \times 4\pi [R^2 - (r_1^2 + r_2^2)]$$
$$= 435.5 \times 10^{-3} \times 4 \times 3.14 [(2.1 \times 10^{-3})^2 - (1 \times 10^{-6} + 4 \times 10^{-6})]$$
$$= 435.5 \times 4 \times 3.14 [4.41 - 5] \times 10^{-6} \times 10^{-3}$$
$$= -32.27 \times 10^{-7} = -3.22 \times 10^{-6} \text{ J} \quad [1]$$

(Negative sign shows absorption)

Therefore, 3.22×10^{-6} J energy will be absorbed.

16. *If a drop of liquid breaks into smaller droplets, it results in lowering of temperature of the droplets. Let a drop of radius R, break into N small droplets each of radius r. Estimate the lowering in temperature.* [NCERT]

Sol. When a big drop of radius R, break into N droplets each of radius r, the volume remains constant.

∴ Volume of big drop = $N \times$ Volume of small drop

$$\frac{4}{3}\pi R^3 = N \times \frac{4}{3}\pi r^3$$

or $R^3 = Nr^3$ or $N = \frac{R^3}{r^3}$ [1]

Now, change in surface area = $4\pi R^2 - N4\pi r^2$

$$= 4\pi (R^2 - Nr^2)$$

Energy released = $S \times \Delta A = S \times 4\pi(R^2 - Nr^2)$ [1]

Due to releasing of this energy, the temperature is lowered.

If ρ is the density and s is specific heat of liquid and its temperature is lowered by $\Delta\theta$ then,

Energy released = $ms\Delta\theta$

$$S \times 4\pi (R^2 - Nr^2) = \left(\frac{4}{3} \times R^3 \times \rho\right) s\Delta\theta$$

$$\Delta\theta = \frac{S \times 4\pi (R^2 - Nr^2)}{\frac{4}{3}\pi R^3 \rho \times s} = \frac{3S}{\rho s}\left[\frac{R^2}{R^3} - \frac{Nr^2}{R^3}\right]$$

$$= \frac{3S}{\rho s}\left[\frac{1}{R} - \frac{(R^3/r^3) \times r^2}{R^3}\right] \quad [1]$$

$$\Delta\theta = \frac{3S}{\rho s}\left[\frac{1}{R} - \frac{1}{r}\right]$$

17. *What is the pressure inside the drop of mercury of radius 3.00 mm at room temperature? Surface tension of mercury at that temperature (20°C) is 4.65×10^{-1} N/m. The atmospheric pressure is 1.01×10^5 Pa. Also give the excess pressure inside the drop.*

Excess pressure inside a liquid drop is given by $\Delta p = \frac{2S}{R}$, where, S = surface tension of the liquid, R = radius of the drop. [NCERT]

Sol. Given, radius of drop (R) = 3.00 mm = 3×10^{-3} m

Surface tension of mercury (S) = 4.65×10^{-1} N/m

Atmospheric pressure (p_0) = 1.01×10^5 Pa [1]

Pressure inside the drop = Atmospheric pressure + Excess pressure inside the liquid drop = $p_0 + \frac{2S}{R}$

$$= 1.01 \times 10^5 + \frac{2 \times 4.65 \times 10^{-1}}{3 \times 10^{-3}}$$
$$= 1.01 \times 10^5 + 3.10 \times 10^2$$
$$= 1.01 \times 10^5 + 0.00310 \times 10^5$$
$$= 1.01310 \times 10^5 \text{ Pa} \quad [1]$$

Excess pressure inside the drop

$$(\Delta p) = \frac{2S}{R} = \frac{2 \times 4.65 \times 10^{-1}}{3 \times 10^{-3}}$$
$$= 3.10 \times 10^2 = 310 \text{ Pa} \quad [1]$$

18. *A U-shaped wire is dipped in a soap solution and removed. The thin soap film formed between the wire and a light slider supports a weight of 1.5×10^{-2} N (which includes the small weight of the slider). The length of the slider is 30 cm. What is the surface tension of the film?* [NCERT]

Sol. Length of the slider (l) = 30 cm

As a soap film has two free surfaces, therefore total length of the film to be supported

$$l' = 2l = 2 \times 30$$
$$= 60 \text{ cm} = 0.60 \text{ m} \quad [1]$$

Let S be the surface tension of the soap solution.

Total force on the slider due to surface tension

$$F = S \times 2l$$
$$F = S \times 0.60 \text{ N} \quad \ldots(i)$$

Weight (w) supported by the slider = 1.5×10^{-2} N [1]

In equilibrium,

Force on the slider due to surface tension = weight supported by the slider

∴ $F = w$

$$S \times 0.60 = 1.5 \times 10^{-2}$$

or $S = \frac{1.5 \times 10^{-2}}{0.60}$

$$= 2.5 \times 10^{-2} \text{ N/m} \quad [1]$$