PROGRESSIVE WAVE

1. *A transverse harmonic wave on a string is described as*

$y(x, t) = 3.0 \sin(36t + 0.018x + \pi/4)$

where x, y are in cm and t in s. The positive direction of x is from left to right.

(i) *Is this a travelling wave or a stationary wave ? If it is travelling, what are the speed and direction of its propagation ?*

(ii) *What are its amplitude and frequency ?*

(iii) *What is the initial phase at the origin ?*

(iv) *What is the least distance between two successive crests in the wave ?*

Sol. We have the equation of harmonic wave as

$$y(x, t) = A \sin\left\{\frac{2\pi}{\lambda}(x - vt) + \phi\right\} \qquad ...(i)$$

while the given equation is

$$y(x, t) = 3.0 \sin(0.018x + 36t + \pi/4) \qquad ...(ii)$$

Comparing equations (i) and (ii), we get following information.

(i) It is a travelling wave. The wave is travelling from right to left since the sign of the term containing '*t*' is positive.

Now, comparing the coefficients of x and t terms in equations (i) and (ii), we get

$$\frac{2\pi}{\lambda} = 0.018 \text{ and } \frac{2\pi v}{\lambda} = 36.$$

Dividing, we get $v = \frac{36}{0.018} = 2000 \text{ cm/s} = 20 \text{ m/s}.$

Thus, *the wave is travelling with a speed of 20 m/s from right to left.* **Ans.**

(ii) *Amplitude is* 3.0 cm. **Ans.**

Since $\frac{2\pi}{\lambda} = 0.018$ [See (i) above]

$$\lambda = \frac{2\pi}{0.018} = \frac{2 \times 3.1412}{0.018}$$
$$= 348.9 \text{ cm} = 3.489 \text{ m}$$

Also $\nu = \frac{v}{\lambda} = \frac{20}{3.489} = 5.7 \text{ s}^{-1}.$

∴ *The frequency of the wave,* ν = **5.7 Hz. Ans.**

(iii) To calculate initial phase at the origin, we put $x = 0$ (at the origin) and $t = 0$ (initial) in the argument of equation (2).

∴ *The initial phase is* $\pi/4$ **Ans.**

(iv) *The distance between successive crests = wavelength*

= 348.9 cm (from ii) = **3.5 m. Ans.**

2. The equation of a wave travelling in x-direction on a string is

$y = (3.0 \text{ cm}) \sin[(3.14 \text{ cm}^{-1})x - (314 \text{ s}^{-1})t].$

(a) Find the max. velocity of a particle of the string.

(b) Find the acceleration of a particle at x = 6 cm and a time t = 0.11 s.

Ans. $y = 3 \sin[3.14x - 314t]$

$$v = \frac{dy}{dt} = 3\cos[3.14x - 314t] \times 314$$

$$v_{max} = 3 \times 314 \times 1 \text{ cm/s} = 9.4 \text{ m/s}$$

(b) $$a = \frac{dv}{dt} = -3\sin[3.14x - 314t] \times (314)^2$$
$$= -3(314)^2 \sin(6\pi - 11\pi) = \text{Zero}.$$

3. A travelling harmonic wave is given as

$y = 2.0 \cos(10t - 0.0080x + 0.18)$

where x and y are in cm and t is in s. What is the phase difference between two points separated by

(i) distance of 0.5 m (ii) time gap of 0.5 s?

Ans. Comparing with standard equation

$$y = r\cos\left[\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi_0\right],$$

r = 2.0 cm, $\frac{2\pi}{T} = 10, T = \frac{2\pi}{10} = \frac{\pi}{5}$ sec.

$$\frac{2\pi}{\lambda} = 0.0080, \ \lambda = 2\pi/0.0080 \text{ cm}$$

(i) Phase difference $= \frac{2\pi}{\lambda} \times$ distance
$= 0.0080 \times 0.5 \times 100$
$= 0.4$ rad

(ii) Phase difference $= \frac{2\pi}{T} \times \text{time} = 10 \times 0.5$
$= 5$ rad

4. A progressive wave of frequency 500 Hz is travelling with a velocity of 360 m/s. How far apart are two points 60° out of phase?

Ans. $$\lambda = \frac{v}{\nu} = \frac{360}{500} = 0.72 \text{ m}.$$

As $$\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x$$

∴ $$\Delta x = \frac{\lambda}{2\pi}\Delta\phi = \frac{0.72}{2\pi} \times \frac{\pi}{3} = 0.12 \text{ m}$$

5. *A wave travelling along a string is given by,*
$y(x, t) = 0.005 \sin (80.0\, x - 3.0\, t)$,
in which the numerical constants are in SI units. Calculate
(a) the amplitude, (b) the wavelength, and (c) the period and frequency of the wave.
Also calculate the displacement y of the wave at a distance x = 30.0 cm and time t = 20 s ?

Sol. On comparing with
$y(x, t) = a \sin (kx - \omega t)$,

(a) the amplitude of the wave is 0.005 m = 5 mm.

(b) $k = 80.0\ \text{m}^{-1}$ and $\omega = 3.0\ \text{s}^{-1}$

$$\lambda = 2\pi/k = \frac{2\pi}{80.0\,\text{m}^{-1}} = 7.85\ \text{cm}$$

(c) $$T = 2\pi/\omega = \frac{2\pi}{3.0\,\text{s}^{-1}} = 2.09\ \text{s}$$

$f = 1/T = 0.48$ Hz

The displacement y at x = 30.0 cm and time t = 20 s is given by

$y = (0.005\ \text{m}) \sin (80.0 \times 0.3 - 3.0 \times 20)$
$= (0.005\ \text{m}) \sin (-36 + 12\pi)$
$= (0.005\ \text{m}) \sin (1.699)$
$= (0.005\ \text{m}) \sin (97^0) = 5$ mm (approx.)

6. *For the travelling harmonic wave*
$y(x, t) = 2.0 \cos 2\pi(10t - 0.0080x + 0.35)$
where x and y are in cm and t in s.

Calculate the phase difference between oscillatory motion at two points separated by a distance of (i) 4m, (ii) 0.5 m, (iii) $\frac{\lambda}{2}$ (iv) $\frac{3\lambda}{4}$

Sol. $y(x, t) = 2.0 \cos 2\pi\,(10t - 0.08x + 0.35)$

The whole angle $2\pi(10t - 0.008x + 0.35)$ is a measure of phase. So, phase difference between two points having $x = x_1$ and x_2 will be given by

$= 2\pi \times 0.008\, (x_1 \sim x_2)$

where x is measured in cm (as given in the question) and $(x_1 - x_2)$ is called path difference.

(*i*) $x_1 - x_2$ is given as 4 m *i.e.* 400 cm

$\therefore$ Phase diff. $= 2\pi \times 0.08\, (400) = 6.4\pi$ rad. **Ans.**

(*ii*) Reqd. phase diff. $= 2\pi \times 0.008 \times (0.5 \times 100)$
$= 0.8\pi$ rad. **Ans.**

(*iii*) Here, path difference is given in terms of λ, hence there is no need of the given wave eqn. to find out the required phase difference. It is because

A path diff. of $\lambda \equiv$ A phase diff. of 2π

$\therefore$ A path diff. of $x \equiv$ A phase diff. of $\frac{2\pi x}{\lambda}$

$\therefore$ A path diff. of $\lambda/2 \equiv$ A phase diff. of $\frac{2\pi(\lambda/2)}{\lambda}$

$= \pi$ rad. **Ans.**

(*iv*) Similarly,
a path diff. of $3\lambda/4$
$\equiv$ A phase diff. of $\frac{2\pi(3\lambda/4)}{\lambda} = 3\pi/2$ rad.

Now, students should note that a phase difference of 2π is equal to zero phase difference *i.e.* a phase difference of θ is same as a phase difference of $(2\pi - \theta)$. Therefore, a phase difference of $3\pi/2$ can be written as a phase difference of $(2\pi - 3\pi/2)$ *i.e.* $\pi/2$ rad.

Hence, customarily, the magnitude of phase is given between 0 and 2π and that of phase difference is given between 0 and π.

$\therefore$ Reqd. phase difference $= \pi/2$ rad.

STATIONARY WAVE

7. *The transverse displacement of a string (clamped of its two ends) is given by*

$$y(x, t) = 0.060 \sin \left(\frac{2\pi}{3} x\right) \cos (120\, \pi t)$$

where x and y are in m and t is in s. The length of the string is 1.5 m and its mass is 3.0×10^{-2} kg. Answer the following :

(a) Does the function represent a travelling wave or a stationary wave ?

(b) Interpret the wave as a superposition of two waves travelling in opposite directions. What are the wavelength, frequency, and speed of each wave?

(c) Determine the tension in the string.

Sol. (*a*) Due to clamping, the waves will be reflected back and forth at the two ends. Hence, *the function represents a stationary wave.*

(*b*) The given equation is

$$y = 0.060 \sin \frac{2\pi}{3} x \cos (120\pi t) \quad ...(i)$$

Now, a stationary wave is represented by

$$y = 2A \sin \frac{2\pi}{\lambda} x \cos \frac{2\pi}{\lambda} vt. \quad ...(ii)$$

Comparing equation (*ii*) with equation (*i*), we get

Wavelength of each wave $\lambda = 3$ m. **Ans.**

Also, $\frac{2\pi v}{\lambda} = 120\pi$ or $v = 60 \times 3 = 180$ m/s.

$\therefore$ *Velocity of each wave* = 180 m/s. **Ans.**

8. *Given below are some functions of x and t to represent the displacement (transverse or longitudinal) of an elastic wave. State which of these represent (i) a travelling wave, (ii) a stationary wave or (iii) none at all.*

(*a*) $y = 2 \cos (3x) \sin (10\, t)$

(*b*) $y = 2\sqrt{x - vt}$

(*c*) $y = 3 \sin (5x - 0.5t) + 4 \cos (5x - 0.5t)$

(*d*) $y = \cos x \sin t + \cos 2x \sin 2t.$

Sol. (*a*) This is a case of *stationary wave* as the equation is similar to that of a stationary wave.

(*b*) The function *does not represent any wave* as it is not a periodic function.

(*c*) The function represents a *travelling harmonic wave.*

(*d*) The function represents *superposition of two stationary waves.*

9. *A travelling harmonic wave on a string is described by*

$$y = 7.5 \sin (0.0050x + 12t + \pi/4)$$

(*a*) *What are the displacement and velocity of oscillation of a point at x = 1 cm, and t = 1 s? Is this velocity equal to the velocity of wave propagation ?*

(*b*) *Locate the points of the string which have the same transverse displacement and velocity as the x = 1 cm point at t = 2 s, 5 s, 11 s.*

Sol. (*a*) Substituting $x = 1$ and $t = 1$ in the given equation of the harmonic wave, we get

$$\begin{aligned} y &= 7.5 \sin (0.0050 + 12 + \pi/4) \\ &= 7.5 \sin (12.56 + 0.225) \\ &= 7.4 \sin (4\pi + 0.225) = 7.5 \sin 0.225 \\ &= 7.5 \times 0.225 = 1.687 = 1.7 \text{ cm.} \end{aligned}$$

$\therefore$ *The displacement at x = 1 cm and t = 1 sec is* given by 1.7 cm. **Ans.**

Differentiating the equation w.r.t. t we get $\frac{dy}{dt}$ which is velocity of oscillation of a point.

Now, $\frac{dy}{dt} = 7.5 \times 12 \cos (0.0050x + 12t + \pi/4)$
$= 7.5 \times 12 \cos (4\pi + 0.225)$

for $x = 1$ and $t = 1$ as calculated above.

$= 7.5 \times 12 \cos (0.225) = 7.5 \times 12 \times 0.976$
$= 87.84 = 88$ cm/s.

$\therefore$ *The velocity of oscillation of the point* $(x = 1)$ = 88 cm/s. **Ans.**

To calculate the *velocity of propagation of the wave*, we compare the given equation with the standard equation

$$y = A \sin \frac{2\pi}{\lambda} (x - vt + \phi).$$

$$\therefore \quad \frac{2\pi}{\lambda} = 0.0050$$

$$\lambda = \frac{2\pi}{0.0050} = 1260 \text{ cm} = 12.60 \text{ m.}$$

and $\frac{2\pi}{\lambda} v = -12$ giving $v = \frac{-12}{0.0050}$
$= -2400$ cm/s $= -24$ m/s.

Thus, *the velocity of oscillation of the point (x = 1)* is 88 cm/s while the *velocity of propagation of the wave* is –24 m/s. Hence, *the two are different.* **Ans.**

(*b*) Now, all points which are at a distance of $\pm \lambda, \pm 2\lambda, \pm 3\lambda$ etc. from the point $x = 1$ will have same transverse displacement and particle velocity, where λ is the wavelength of the wave. Thus, all points which are at distances ± 12.6 m, ± 25.2 m, ± 37.8 m ; ± ....from the point $x = 1$ will have same transverse displacement and velocity as those of the point $x = 1$ cm ($\because \lambda = 12.60$ m).

10. Stationary waves are set up by the superposition of two waves given by

$$y_1 = 0.05 \sin (5\pi t - x)$$

and $$y_2 = 0.05 \sin (5\pi t + x)$$

where x and y are in metre and t in sec. Calculate the amplitude of a particle at a distance of $x = 1$ m.

Ans. Using superposition principle, the resultant displacement at time t is given by

$$y = y_1 + y_2$$

$$= 0.05 \sin(5\pi t - x) + 0.05 \sin(5\pi t + x)$$

$$= 0.05 \times 2 \sin\left(\frac{5\pi t - x + 5\pi t + x}{2}\right) \cos\frac{5\pi t + x - 5\pi t + x}{2}$$

$$= 0.1 \sin 5\pi t \cos x$$

$$y = [0.1 \cos x] \sin 5\pi t$$

Amplitude, $r = 0.1 \cos x$

At $x = 1$ m,

$$r = 0.1 \cos 1 = 0.1 \cos 180°/\pi$$

$$= 0.1 \cos 57.3° = 0.1 \times 0.5406 = 0.054 \text{ m.}$$

11. The distance between two points on a stretched string is 20 cm. A progressive wave of frequency 400 Hz travels on the string with a velocity of 100 m/s. Calculate phase difference between the points.

Ans.

$$\lambda = \frac{v}{\nu} = \frac{100}{400} = 0.25 \text{ m}$$

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{0.25} \times \left(\frac{20}{100}\right)$$

$$= 1.6\,\pi \text{ radian}$$

NEWTON'S FORMULA FOR VELOCITY OF SOUND

12. Speed of sound in air is 332 m/s at N.T.P. What will be its value in hydrogen at N.T.P. if density of hydrogen at N.T.P. is 1/16th that of air?

Ans. As $v = \sqrt{\frac{\gamma P}{\rho}}$, taking γ and P same for air and hydrogen, we get

$$\frac{v_H}{v_a} = \sqrt{\frac{\rho_a}{\rho}} = \sqrt{\frac{\rho_a}{1/16\rho_a}} = 4$$

$$v_H = 4\, v_a = 4 \times 332 = 1328 \text{ m/s.}$$

13. The speed of sound in dry air at N.T.P. is 332 m/s. Assuming air as composed of 4 parts of nitrogen and one part of oxygen, calculate velocity of sound in oxygen under similar conditions, when the densities of oxygen and nitrogen at N.T.P. are in the ratio of 16 : 14 respectively.

Ans.

$$\frac{\rho_0}{\rho_n} = \frac{16}{14}$$

Let volume of oxygen in air = V

∴ Volume of nitrogen in air = 4V

Total volume of air = V + 4V = 5V

Mass of oxygen in air = V × 16 = 16V

Mass of nitrogen in air = 4V × 14 = 56V

Total mass of air = 16 V + 56V = 72V

$$\therefore \text{ density of air, } \rho_a = \frac{\text{total mass of air}}{\text{total volume of air}}$$

$$\rho_a = \frac{72V}{5V} = 14.4$$

$$\text{As } \frac{v_0}{v_a} = \sqrt{\frac{\rho_a}{\rho_0}}$$

$$\therefore \frac{v_0}{332} = \sqrt{\frac{14.4}{16}} = \sqrt{0.9} = 0.9487$$

$$\therefore \quad v_0 = 332 \times 0.9487 = 314.77 \text{ ms}^{-1}$$

14. If the splash is heard 4.23 s after a stone is dropped into a 78.4 m deep well, find the velocity of sound in air.

Ans. Depth of the well, $s = 78.4$ m

Total time after which splash is heard = 4.23 s

If t_1 = time taken by stone to hit the water surface in the well,

t_2 = time taken by splash of sound to reach the top of the well,

then $t_1 + t_2 = 4.23$ s

For downwards journey of stone,

$u = 0$, $a = 9.8$ ms^{-2}, $s = 78.4$ m

$$\text{As} \quad s = ut + \frac{1}{2}at^2$$

$$\therefore \quad 78.4 = 0 + \frac{1}{2} \times 9.8\, t_1^2 = 4.9\, t_1^2$$

$$\text{or} \quad t_1^2 = \frac{78.4}{4.9} = 16 \quad \text{or} \quad t_1 = 4 \text{ s}$$

From $t_1 + t_2 = 4.23$

$$t_2 = 4.23 - t_1 = 4.23 - 4 = 0.23 \text{ s}$$

$$v = \frac{\text{distance}(s)}{\text{time}(t_2)} = \frac{78.4}{0.23} = 340.87 \text{ ms}^{-1}$$

What is the speed of a transverse wave in a rope of length 30 m and mass 0.09 kg under a tension of 270 N?

Ans. $m = \frac{0.09}{30} = 3 \times 10^{-3}$ kg/m, T = 270 N

$v = \sqrt{T/m} = 300 \text{ ms}^{-1}$.

15. A tuning fork of frequency 220 Hz produces sound waves of wavelength 1.5 m in air at N.T.P. Calculate the increase in wavelength, when temperature of air is increased to 27°C.

Ans. $v_0 = \nu \lambda_0 = 220 \times 1.5 = 330$ m/s

$$v_t = v_0\sqrt{\frac{T}{T_0}} = 330\sqrt{\frac{273+27}{273}} = 346 \text{ ms}^{-1}$$

$$\lambda_1 = \frac{v_t}{\nu} = \frac{346}{220} = 1.568 \text{ m}$$

$\lambda_1 - \lambda_0 = 1.568 - 1.5 = 0.068$ m.

16. *Given below are some examples of wave motion. State in each case if the wave motion is transverse, longitudinal or a combination of both :*

(a) Motion of a kink in a long coil spring produced by displacing one end of the spring sideways.

(b) Waves produced in a cylinder containing a liquid by moving its piston back and forth.

(c) Waves produced by a motorboat sailing in water.

(d) Ultrasonic waves in air produced by a vibrating quartz crystal.

Sol. (a) Transverse

(b) Longitudinal

(c) Transverse and longitudinal

(d) Longitudinal

17. *Estimate the speed of sound in air at standard temperature and pressure. The mass of 1 mole of air is 29.0×10^{-3} kg.*

Sol. Density of air at STP is :

ρ_o = (mass of one mole of air) / (volume of one mole of air at STP)

$$= \frac{29.0 \times 10^{-3} \text{ kg}}{22.4 \times 10^{-3} \text{ m}^3} = 1.29 \text{ kg m}^{-3}$$

According to Newton's formula,

$$v = \sqrt{\frac{P}{\rho}} = \left[\frac{1.01 \times 10^5 \text{ N m}^{-2}}{1.29 \text{ kg m}^{-3}}\right]^{1/2} = 280 \text{ m s}^{-1}$$

18. *A stone dropped from the top of a tower height 300 m high splashes into the water of a pond near the base of the tower. When is the splash heard at the top? Given that the speed of sound in air is 340 m s^{-1} [$g = 9.8$ m s^{-2}].*

Sol. The time t_1 taken by the stone to reach the pond, is given by

$$S = ut_1 + \frac{1}{2}gt_1^2. \qquad ...(i)$$

Here $S = 300$ m, $u = 0$ and $g = 9.8$ m/s². Substituting the values in equation (1), we get

$$300 = \frac{1}{2} \times 9.8 \times t_1^2$$

or

$$t_1 = \sqrt{\frac{300}{4.9}} = 7.82 \text{ s.}$$

If the time taken by the sound of the splash to reach the top of the tower is t_2, it is given by the relation

$S = vt_2$, where v is the speed of sound in air

or $$t_2 = \frac{S}{v} = \frac{300}{340} = 0.88 \text{ s.}$$

∴ *The time after which the splash is heard* = $t_1 + t_2 = 7.82 + 0.88 =$ **8.7 s Ans.**

19. *A bat emits ultrasonic sound of frequency 1000 kHz in air. If the sound meets a water surface, what is the wavelength of (a) the reflected sound, (b) the transmitted sound ? Speed of sound in air 3.40 m s^{-1} and in water 1486 m s^{-1}.*

Sol. Whenever a wave meets a surface separating two media, part of it is reflected and a part is transmitted. The frequency does not change.

(*a*) For the reflected wave in air

Here $v = 340 \text{ ms}^{-1}$ and $\nu = 100 \text{ kHz} = 10^5$ Hz

$$\therefore \quad \lambda = \frac{v}{\nu} = \frac{340}{10^5} = 340 \times 10^{-5}$$

$$= 3.40 \times 10^{-3} \text{ m. } \textbf{Ans.}$$

20 . A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency of 45 Hz. The mass of the wire is 3.5×10^{-2} kg and its linear density is 4×10^{-2} kg/m. What is the speed of transverse waves on string and tension in the wire?

Ans. Length of wire $= \dfrac{\text{mass of wire}}{\text{linear density}}$

$$= \frac{3.5\times10^{-2}}{4\times10^{-2}} = 0.875 \text{ m}$$

$$\nu = \frac{1}{2l}\sqrt{\frac{T}{m}} \quad , \quad T = 4\nu^2 l^2 m$$

$$T = 4 \times (45)^2 \times (0.875)^2 \times 4 \times 10^{-2} = 248.1 \text{ N}$$

Speed of transverse waves,

$$v = \nu\lambda = \nu(2l) = 45 \times 2 \times 0.875 = 78.75 \text{ m/s}$$

21 . A stretched wire emits a fundamental note of 256 Hz. Keeping the stretching force constant and reducing the length of the wire by 10 cm, the frequency becomes 320 Hz. Calculate the original length of the wire.

Ans. The frequency of fundamental note is

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}}$$

In the 1st case, $256 = \dfrac{1}{2L}\sqrt{\dfrac{T}{m}}$ ...(i)

In the 2nd case, $320 = \dfrac{1}{2(L-10)}\sqrt{\dfrac{T}{m}}$ ...(ii)

Dividing (i) by (ii), we get $\dfrac{256}{320} = \dfrac{(L-10)}{L}$

On solving, L = 50 cm.

22 . The length of a sonometer wire between two fixed ends is 110 cm. Where should the two bridges be placed so as to divide the wire into three segments, whose fundamental frequencies are in the ratio 1 : 2 : 3?

Ans. Total length of the wire, L = 110 cm

$$\nu_1 : \nu_2 : \nu_3 = 1 : 2 : 3$$

If l_1, l_2, l_3 are the lengths of these three parts, then as $\nu \propto \dfrac{1}{l}$

$$\therefore \quad l_1 : l_2 : l_3 = \frac{1}{1} : \frac{1}{2} : \frac{1}{3} = 6 : 3 : 2$$

Sum of the ratio $= 6 + 3 + 2 = 11$

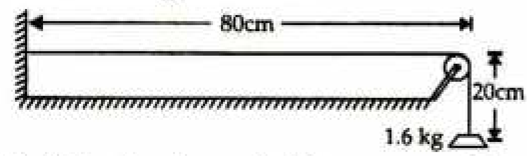
$$\therefore \quad l_1 = \frac{L}{11} \times 6 = \frac{110}{11} \times 6 = 60 \text{ cm}$$

$$l_2 = \frac{L}{11} \times 3 = \frac{110}{11} \times 3 = 30 \text{ cm}$$

$$l_3 = \frac{L}{11} \times 2 = \frac{110}{11} \times 2 = 20 \text{ cm}$$

Hence the bridges should be placed at 60 cm and 90 cm from the zero end of the wire.

23 . A 100 cm long wire of mass 40 g supports a mass of 1.6 kg as shown.



Find the fundamental frequency of the portion of the string between the wall and the pulley. Take $g = 10 \text{m/s}^2$.

Ans. Here, $T = mg = 1.6 \times 10 = 16$ N

$$\text{mass/length, } m = \frac{40\times10^{-3}\text{kg}}{1\text{m}}$$

$$= 0.04 \text{ kg/m.}$$

$$\lambda = 100 - 20 = 80 \text{ cm} = 0.8 \text{ m}$$

$$\nu = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2\times0.8}\sqrt{\frac{16}{0.04}} = 12.5 \text{ Hz}$$

24 . If the fundamental frequency of an organ pipe is 150 Hz, what is the frequency of

(a) 2nd harmonic if it is closed at one end,

(b) second harmonic if it is open at both ends,

(c) second overtone if it is closed at one end,

(d) second overtone if it is open at both ends.

Ans.(a) In a pipe closed at one end, second harmonic does not exist.

(b) $\nu_2 = 2\nu_1 = 2 \times 150 = 300$ Hz.

(c) In a pipe closed at one end, frequency of second overtone $= 5\nu_1 = 5 \times 150 = 750$ Hz.

(d) In an open pipe, 2nd overtone is 3rd harmonic of frequency $= 3\nu_1 = 3 \times 150 = 450$ Hz.

25 . An air column is constructed by fitting a movable piston in a long cylindrical tube.

Longitudinal waves are sent in the tube by a tuning fork of frequency 416 Hz. How far from the open end should the piston be so that the air column in the tube may vibrate in its first overtone?

Speed of sound in air is 333 m/s.

Ans. Here, $\lambda = \frac{v}{\nu} = \frac{333}{416} = 0.8\text{ m}$

For vibrating in first overtone mode,

$$L = \frac{3\lambda}{4} = \frac{3\times 0.8}{4} = 0.6\text{ m}$$

26 . A 30 cm long pipe is open at both ends. Which harmonic mode of the pipe is resonantly excited by a 1.1 kHz source? Will resonance with the same source be observed if one end of the pipe is closed? Take the speed of sound in air as 330 m/s.

Ans. In case of open pipe, fundamental frequency = frequency of 1st harmonic

$$\nu_1 = \frac{v}{2l} = \frac{330}{2\times 0.30} = 550\text{ Hz.}$$

The frequencies of 2nd harmonic, 3rd harmonic, 4th harmonic and so on are

$2 \times 500\text{ Hz} = 1100\text{ Hz}$;

$3 \times 550\text{ Hz} = 1650\text{ Hz}$;

$4 \times 550\text{ Hz} = 2200\text{ Hz}$ and so on.

Hence the source of frequency 1.1 kHz *i.e.* 1100 Hz will resonantly excite second harmonic.

If one end of the pipe is closed, the fundamental frequency becomes

$$\nu_1' = \frac{v}{4l} = \frac{330}{4\times 0.30} = 275\text{ Hz}$$

and only odd harmonics are present *i.e.* harmonics have frequencies

$3\nu_1' = 3 \times 275 = 825\text{ Hz}$;

$5\nu_1' = 5 \times 275 = 1375\text{ Hz}$

and so on.

Since no frequency matches with the source frequency, no resonance will be observed with the given source when the pipe is closed.

27 . A tuning fork of unknown frequency gives 4 beats with a tuning fork of frequency 310 Hz. It gives the same number of beats on filing. Find the unknown frequency.

Ans. Out of the two possible frequencies (310 ±4), one is initial value and the other is final value. As frequency increases on filling, therefore initial frequency = 306 Hz.

28 . Two tuning forks A & B produce 4 beats/s. On loading B with wax, 6 beats/s are heard. If quantity of wax is reduced, the number of beats per second again becomes 4. Find the frequency of B if the frequency of A is 256 Hz.

Ans. Possible frequencies of B are $256 \pm 4 = 260$ or 252. As some wax continues to be attached finally and number of beats/s = 4, therefore, final frequency must be less than the initial frequency. Hence initial frequency of B = 260 Hz.

29. 64 tuning forks are arranged in order of increasing frequency. The frequency of the last fork is double of the first and each two consecutive forks produce 4 beats/ second. Calculate the frequency of the last fork.

Ans. Let frequency of 1st fork = ν

frequency of 2nd fork = $\nu + 4(2-1) = \nu + 4$

$\therefore$ frequency of 64th fork = $\nu + 4(64-1) = 2\nu$

or $\nu = 252\text{ Hz}$.

Frequency of last fork

$= 2\nu = 2 \times 252 = 504\text{ Hz}$

30 . A tuning fork of frequency 200 Hz is in unison with a sonometer wire. How many beats/s will be heard if tension in the wire is increased by 2%?

Ans. Let $T_1 = 100$ units

$\therefore$ $T_2 = 102$ units

$$\nu_2 = \nu_1\sqrt{\frac{T_2}{T_1}} = 200\sqrt{\frac{102}{100}}$$

$$= 200\left(1+\frac{2}{100}\right)^{1/2}$$

$$= 200\left(1+\frac{1}{2}\times\frac{2}{100}\right) = 202$$

$\therefore$ No. of beats/s = $\nu_2 - \nu_1 = 202 - 200 = 2$

31. *A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod are given to be 2.53 kHz. What is the speed of sound in steel ?*

Sol. For the rod clamped in the middle, the mid-point is a node and the two free ends are antinodes. Thus when set into vibrations, its length is the distance between consecutive antinodes *i.e.*, $\lambda/2$, thus

$$\lambda/2 = 100 \text{ cm or } \lambda = 2 \text{ m},$$

and $\nu = 2.53 \text{ kHz} = 2.53 \times 10^3 \text{ Hz}$

Therefore $v = \nu\lambda = 2.53 \times 10^3 \times 2$
$= 5.06 \times 10^3 \text{ m s}^{-1}$ or 5.06 km/s. **Ans.**

32. *A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 430 Hz source ? Will the same source be in resonance with the pipe if both ends are open ? (Speed of sound in air is 340 m/s).*

Sol. (*a*) For resonance in case of a closed end organ pipe,

$$L = \frac{(2n-1)\lambda_n}{4} \quad ...(I)$$

For the case given in question

$v = 340 \text{ m/s}$

$\nu = 430 \text{ Hz} \quad \therefore \quad \lambda_n = \frac{340}{430} = 0.8 \text{ m}$

Also L = length of pipe $= 20 \text{ cm} = 0.2 \text{ m}$

Substituting above values in (i)

$$0.2 = \frac{(2n-1)0.8}{4} \quad \text{On solving, } n = 1$$

i.e. the pipe will resonate in first harmonic mode.

(b) Now if the same pipe is now made open at both ends, then for resonance, $L = \frac{n\lambda_n}{2}$

Using the above calculated data,

$$0.2 = \frac{n \times 0.8}{2} \Rightarrow n = 0.5$$

But n should be an integer for resonance, hence, no resonance will occur in case the pipe is an open end pipe.

33. *Two sitar strings A and B playing the note 'Ga' are slightly out of tune and produce beats of frequency 6 Hz. The tension in the string A is slightly reduced and the beat frequency is found to reduce to 3 Hz. If the original frequency of A is 324 Hz, what is the frequency of B ?*

Sol. If n_A is original frequency of the wire A and n_B is the frequency of the wire B, we have $n_B = n_A \pm 6 = 324 \pm 6$.

i.e., $n_B = 330 \text{ Hz or } 318 \text{ Hz}$.

When the tension in A is reduced, its frequency is also reduced because

$$n_A = \frac{1}{2l}\sqrt{\frac{T}{m}}.$$

Now, the number of beats becomes 3.

$\therefore$ *The frequency of B is* **318 Hz. Ans.**

34. *Explain why (or how):*

(*a*) *In a sound wave, a displacement node is a pressure antinode and vice versa,*

(*b*) *Bats can ascertain distances, directions, nature and sizes of the obstacles without any "eyes",*

(*c*) *A violin note and sitar note may have the same frequency, yet we can distinguish between the two notes,*

(*d*) *Solids can support both longitudinal and transverse waves, but only longitudinal waves can propagate in gases.*

Sol. (*a*) At the nodes the displacement is zero but strain is maximum. Thus, *at the nodes pressure is maximum*. Therefore, a displacement node is a pressure anti-node.

(*b*) Bats can produce ultrasonic waves and receive them after reflection. From the time delay between production of ultrasonic pulse and its reception back, bats can estimate the size, direction etc. of the obstacles which reflect the ultrasonic waves.

(*c*) Though the notes are of the same frequency, yet the overtones and their relative strength is different in the two notes. As a result, timber of the two notes are different. Hence, the two notes can be distinguished.

(*d*) For propagation of longitudinal waves, a medium should possess bulk modulus, longitudinal waves can be propagated in all the three media. On the other hand, for the propagation of transverse waves, a medium should possess Young's modulus and coefficient of rigidity. Since gases do not have Young's modulus and coefficient of rigidity, transverse waves cannot propagate in gases.

35 . *A string of mass 2.50 kg is under a tension of 200 N. The length of the stretched string is 20.0 m. If a transerse jerk is struck at one end of the string, how long does the disturbance take to reach the other end?*

Sol. Here, Tension T = 200 N

mass of string = 2.50 kg

length of string = 20 m

Hence m = mass/length = $\frac{2.5}{20}$ kg/m.

Using the formula for speed of transverse waves, on substitution, we get

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{200}{2.5/20}} = \sqrt{\frac{200 \times 20}{2.5}} = 40 \text{ m/s}$$

Thus time taken by the disturbance to cover a distance of 20 m is

$$t = \frac{\text{Distance}}{\text{Speed}} = \frac{20}{40} = \mathbf{0.5\ s.\ Ans.}$$

36 . *A steel wire has a length of 12.0 m and a mass of 2.10 kg. What should be the tension in the wire so that the speed of a transverse wave on the wire equals the speed of sound in dry air at 20° C (= 343 ms^{-1}) ?*

Sol. Here mass of the wire = 2.10 kg

length of wire = 12.0 m

$\therefore \quad m = \frac{2.10}{12.0}$ kg/m = 0.175 kg/m

Now, it is required to be given a tension such that the speed of the wave is 343 ms^{-1}. *i.e.* v = 343 ms^{-1}

Using formula, $v = \sqrt{T/m}$ or $T = v^2 m$

we get, $T = (343)^2 \times 0.175$

$= \mathbf{2.06 \times 10^4\ N.\ Ans.}$

37 . *A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency of 45 Hz. The mass of the wire is 3.5 × 10^{-2} kg and its linear density is 4.0 × 10^{-2} kg/m. What is (a) the speed of a transverse wave on the string, and (b) the tension in the string ?*

Sol. (*a*) Since the wire is vibrating in the fundamental mode, its length $l = \frac{\lambda}{2}$

or $\lambda = 2 \times \text{length} = 2 \times \frac{\text{Mass}}{\text{Mass per unit length}}$

$$= \frac{2 \times 3.5 \times 10^{-2}}{4.0 \times 10^{-2}} = 1.75 \text{ m.}$$

$\therefore$ The speed of the wave,

$v = \nu\lambda = 45 \times 1.75 = 78.75$ m/s

[$\because \nu = 45$ Hz, given]

(*b*) *The tension in the string,*

$T = v^2 \times m \qquad [\because v = \sqrt{\frac{T}{m}}]$

$= (78.75)^2 \times 4.0 \times 10^{-2}$

$= 248.06 = \mathbf{248.1\ N\ Ans.}$

38 . *A metre-long tube open at one end, with a movable piston at the other end, shows resonance with a fixed frequency source (a tuning fork of frequency 340 Hz) when the tube length is 25.5 cm or 79.3 cm. Estimate the speed of sound in air at the temperature of the experiment. Ignore edge effect.*

Sol. Since there is a piston at one end of the tube, it will behave as a closed organ pipe. We know that a closed pipe always produces odd harmonics. Also, the other resonating length (79.3 cm) is about 3 times the first. Hence, the pipe is in resonance with the fundamental and the third harmonic.

Now, in the fundamental mode, $\frac{\lambda}{4} = l_1 = 25.5$

or $\lambda = 4 \times 25.5 = 102\text{cm} = 1.02$ m.

$\therefore$ *The speed of sound in air*

$= v = \nu\lambda = 340 \times 1.02 = \mathbf{346.8\ m/s.}$

39. *A steel wire is 0.72 m long and has a mass of 5.0 ×10^{-3} kg. If the wire is under a tension of 60 N, what is the speed of transverse waves on the wire?*

Sol. Mass per unit length of the wire,

$$\mu = \frac{5.0 \times 10^{-3}\text{ kg}}{0.72\text{ m}} = 6.9 \times 10^{-3}\text{ kg m}^{-1}$$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{60\text{N}}{6.9 \times 10^{-3}\text{ kg m}^{-1}}} = 93 \text{ ms}^{-1}$$