Wave

WAVE MOTION
It is a periodic disturbance through which energy and momentum is transferred from one point to another without the transfer of material medium.

CHARACTERISTICS OF WAVE MOTION.

(i) In wave motion, the phase of particles of medium keeps on changing.

(ii) The velocity of the particles during their vibration is different at different position.

(iii) The velocity of wave motion through a particular medium is constant. It depends only on the nature of medium not on the frequency, wavelength or intensity.

(iv) Energy is propagated along with the wave motion without any net transport of the matter.

(v) For the propagation of wave, a medium should have following characteristics,

Elasticity: So that particles can return to their mean position, after having been displaced from there.

Inertia : So that particles can store energy and overshoot their mean position.

Minimum friction amongst the particles of the medium.

Uniform density of the medium**.**

TYPES OF WAVE.

Mechanical wave. Mechanical wave require material medium for propagation, like water, sound, seismic waves etc.

Non-mechanical wave. Non-mechanical wave do not require any material medium for propagation like electromagnetic waves e.g. heat waves, light waves, X-rays etc.

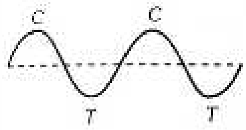
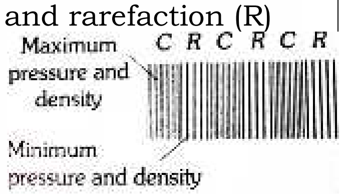
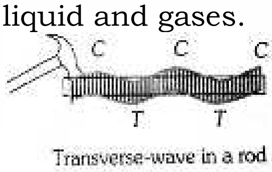
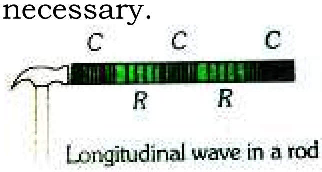
Matter Waves: Matter waves are the waves in which matter i.e. electrons, protons, neutrons, atoms or molecules move. Electron waves are used in electron microscope.

Shock waves. When an object moves with a velocity greater than that of sound, it is termed as Supersonic. When such a a supersonic body or plane travels in air, it produces energetic disturbance which moves in backward direction and diverges in the form of a cone. Such disturbances are called the shock waves.

The speed of supersonic is measured in Mach number. One mach number is the ratio of speed of source to the speed of sound.

Mach Number= $\frac{V \quad of\ S}{V \quad o\ S}$

Transverse and Longitudinal waves

Transverse wave	Longitudinal wave
Particle of the medium vibrates in a direction perpendicular to the direction of propagation of wave.	Particle of the medium vibrates in the direction of wave motion.
It travels in the form of crest (C) and trough (T) 	It travels in the form of compression (C) and rarefaction (R) 
Transverse waves can be transmitted through solids, they can be setup on the surface of liquids. But they cannot be transmitted into liquid and gases. 	These waves can be transmitted through solids, liquids and gases because for propagation these waves propagation, volume elasticity is necessary. 
It posses the property of	It posses the property of elasticity.

rigidity.	
It can be polarised.	It cannot be polarised
Movement of string of a sitar or violin, movement of the membrane of a Tabla or Dholak , wave setup in the surface of water.	Sound wave travel air, vibration of air column in organ pipes vibration of air column above the surface of water in the tube of resonance apparatus.

NEWTON'S FORMULA FOR VELOCITY OF SOUND.

Newton, on the basis of theoretical considerations, deduced the following formula for the velocity of longitudinal waves in an elastic medium.

$$\boxed{v = \sqrt{\frac{E}{\rho}}} \quad \ldots\ldots\ldots \quad (1)$$

Where E=modulus of elasticity-

$\begin{cases} Y - \text{for solid} \\ B - \text{for liquid} \\ \quad \text{and gases} \end{cases}$

Newton's assumed that sound wave travel in air under isothermal conditions, i.e., temperature remains constant. So, the changes in pressure and volume obey Boyle's law. $\therefore$ PV = constant

Differentiating, PdV + VdP= 0

$P = -\frac{d}{d\ /V}$ = B (Bulk modulus of elasticity)

From equation (1), $v = \sqrt{\frac{P}{\rho}} = \sqrt{\frac{0.7 \times 1 \quad \times 9.8}{1.2}} =$ 280 ms^{-1}. This value is nearly 16% less than the experimental value of 332 ms^{-1}.

LAPLACE'S CORRECTION. Laplace's suggested that sound waves travel in air under adiabatic conditions and not under isothermal conditions as suggested by Newton. He gave the following reasons for this.

(i) When sound waves travel in air, the changes in volume and pressure take place rapidly. (ii) Air or gas is a bad conductor of heat.

Due to both these factors, the compressed air becomes warm and stays warm whereas the rarefied air suddenly cools and stays cool. For adiabatic changes in pressure and volume, PV^{γ} = constant

on differentiation,

$$P\gamma V^{\gamma-1}dV + V^{\gamma}dP = 0$$

$P\gamma = -\frac{V^{\gamma}d}{V^{\gamma-1}d} = -\frac{d}{d\ /V} = B$

Using equation (1) $v = \sqrt{\frac{P\gamma}{\rho}} = \sqrt{1.4} \times 280$

= 331.3ms^{-1}.

This result agree very well with experimental value of 332 m/s.

This establishes the correctness of Laplace's formula.

FACTORS AFFECTING THE VELOCITY OF SOUND IN GASES.

1. Effect in change in pressure.

At constant temperature,

PV = const. (Boyle's Law)

$\frac{P}{\rho}$ = cons. or $\frac{P}{\rho}$ = const. (m is constant)

or $\frac{\gamma P}{\rho}$ = const. $\therefore$ v $\left(= \sqrt{\frac{P\gamma}{\rho}}\right)$ is also const.

i.e., velocity of sound is independent of pressure.

2. Effect of change in temperature.

Let v_0 and v_t be the velocity of sound in a gas at 0 ^{0}C and t ^{0}C respectively.

$\frac{v_t}{v_0} = \sqrt{\frac{\rho_0}{\rho_t}}$ .....(1) Here γP is constant.

$\frac{V_t}{V_0} = \frac{T}{T_0}$ (Charle's Law)

$\frac{m/\rho_t}{m/\rho_0} = \frac{T}{T_0}$ $\therefore$ $\frac{\rho_0}{\rho_t} = \frac{T}{T_0}$

Using equation (1), $\boxed{\frac{v_t}{v_0} = \sqrt{\frac{T}{T_0}}}$

In general, $\boxed{v \propto \sqrt{T}}$

Temp coefficient of velocity of sound

$\frac{v_t}{v_0} = \sqrt{\frac{2 \quad + t}{2}} = \left(1 + \frac{t}{2}\right)^{1/2}$

$= \left(1 + \frac{t}{2 \times 2}\right) = \left(1 + \frac{t}{5}\right)$

$\therefore v_t = v_0 + v_0\frac{t}{5} = v_0 + 332\frac{t}{5}$

$= v_0 + 0.61t$ $\boxed{\therefore v_t - v_0 = 0.61t\ ms^{-1}}$

When t = 1^{0}C, then $v_t - v_0 = 0.61ms^{-1}$ so, the velocity of sound increases by 0.61ms^{-1} for every one degree centigrade rise of temperature. This is known as the temperature coefficient of velocity of sound.

3. Effect of change in density.

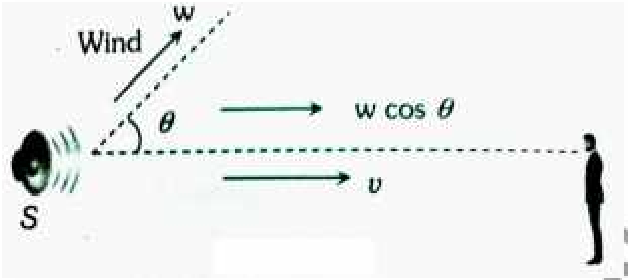
$v = \sqrt{\frac{F_Y}{\rho}}$ i.e., $\mathbf{v} \propto \frac{1}{\sqrt{\rho}}$

4. Effect of humidity : With increase in humidity, density of air decreases. So with rise in humidity velocity of sound increases.

Sound travel faster in humid air (rainy season) than in dry air (summer) at the same temperature because.

$\rho_{m\ a} < \rho_{d\ a}$ □ $v_{m\ a} > v_{d\ a}$

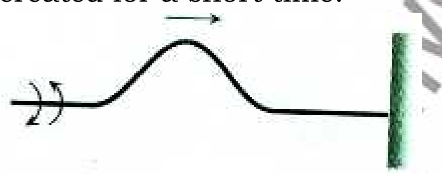
5. Effect of Wind velocity:



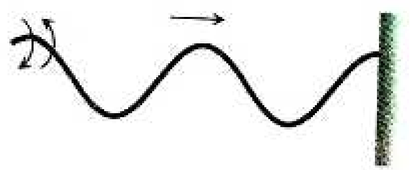
$V_n = v + w\cos\theta$

6. Sound of any frequency or wavelength travels through a given medium with the same velocity.

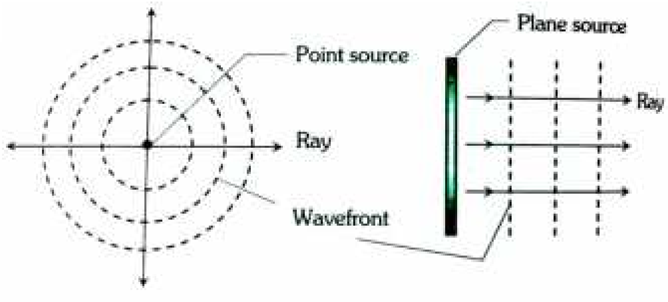
WAVE PULSE : It is a short wave produced in a medium when the disturbance is created for a short time.



WAVE TRAINS : A series of wave pulse is called wave train.



WAVE FRONT : A wave front is a line or surface on which the disturbance has the same phase at all points. If the source is periodic, it produced a succession of wave front, all of the same shape. Ripples on a pond are the example of wave fronts.



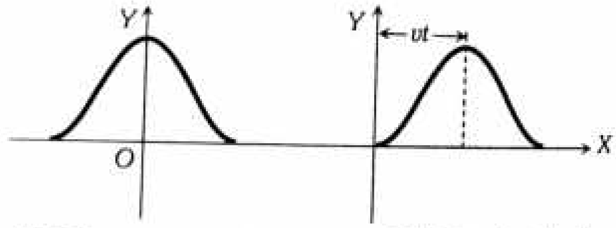
(A) Spherical wavefront **(B)** Plane wavefront

WAVE FUNCTION : It is the mathematical description of the disturbance created by a wave.

Let us consider a one dimensional wave travelling along x-axis. During wave motion , a particle with equilibrium position x is displaced some distance y in the direction perpendicular to the x-axis. In this case y is a function of position (x) and time (t).

i.e. y = f (x, t). This is called wave function.

Let the wave pulse be travelling with a speed v. After a time t, the pulse reaches a distance vt along the +x-axis as shown. The wave function now can be represented as y = f (x – vt)



(A) Pulse at time $t = 0$ **(B)** Pulse after time t

If the wave pulse is travelling along –x-axis then y = f (x + vt).

Thus $y = (x - vt)^2$, $\sqrt{(x - vt)}$, $Ae^{-B(x-v\ )^2}$etc represents travelling waves while

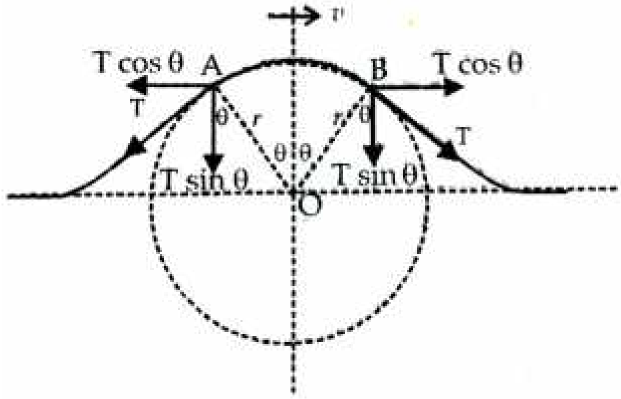
$y = (x^2 - v^2t^2)$, $(\sqrt{x} - \sqrt{vt})$, $A\sin(4x^2 - 9t^2)$etc , do not represent a wave.

HARMONIC WAVE : If a travelling wave ia a function of sin or cos function of (x ± vt) the wave is said to be harmonic or plane progressive wave.

WAVE EQUATION : All the travelling waves satisfy a differential equation which is called the wave equation. It is given by

$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$; where $v = \frac{\omega}{k}$

SPEED OF TRANSVERSE WAVE IN A STRETCHED STRING



Let a wave pulse is proceeding in the string with a speed v and tension T.
Consider a small sector of the pulse, AB.
Δl = Length of AB, which forms the arc of a circle of radius r.
Vertical component of tension = 2Tsinθ
= 2Tθ, If θ is small .... (1)
This vertical component is the centripetal force which provides for the centripetal acceleration of point P directed towards O.
Let m = mass per unit length of the string.

$\therefore$ mass of AB = Δl.m .... (2)

$\therefore 2T\theta = \frac{(m\Delta l).v^2}{r}$ .... (3)

Now , angle = $\frac{a}{r}$

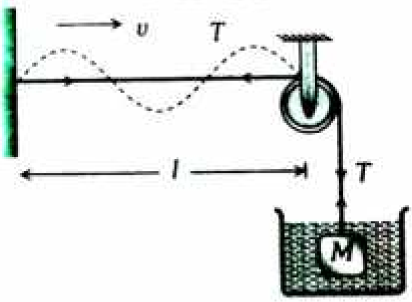
$2\theta = \frac{\Delta l}{r} \Rightarrow \theta = \frac{\Delta l}{2}$ .... (4)

Substituting (4) in (3), we get

$2T\frac{\Delta l}{2} = \frac{m\Delta l.v^2}{r}$

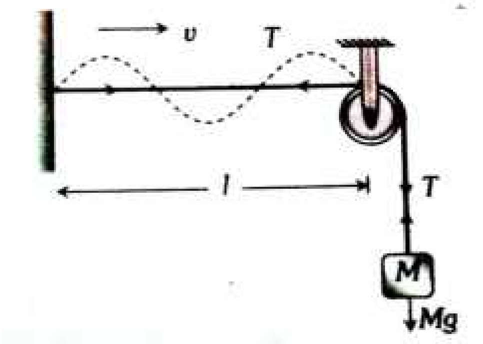
$$\boxed{v = \sqrt{\frac{T}{m}}}$$

(i) If suspended weight is immersed in a liquid of density σ and ρ = density of material of the suspended load then



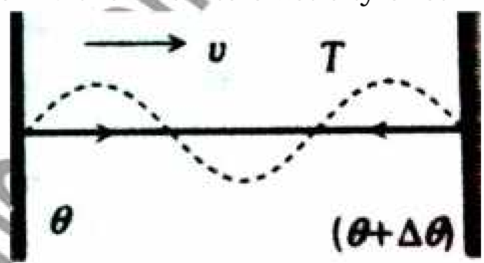
$T = Mg\left(1 - \frac{\sigma}{\rho}\right) \Rightarrow v = \sqrt{\frac{M(1-\sigma/\rho)}{m}}$

(ii) If string is stretched by some weight then



$T = Mg \Rightarrow v = \sqrt{\frac{M}{m}}$

(iii) If two rigid supports of stretched string are maintained at temperature difference of Δθ then due to elasticity of string



$T = YA\alpha\Delta\theta \Rightarrow v = \sqrt{\frac{Y\ \alpha\Delta\theta}{m}} = \sqrt{\frac{Y\alpha\Delta\theta}{d}}$

Y = Young's modulus of elasticity of the string, α = Temperature coefficient of thermal expansion, d = Density of the wire $= \frac{m}{A}$

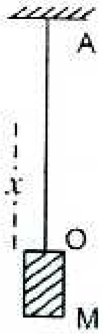
(iv) If A is the area of cross-section of the wire then m = ρA

$\Rightarrow v = \sqrt{\frac{T}{\rho A}} = \sqrt{\frac{S}{\rho}}$, Where S = sress $= \frac{T}{A}$

(v) In a solid body $v = \sqrt{\frac{\eta}{\rho}}$

Where η = Modulus of elasticity, ρ = density

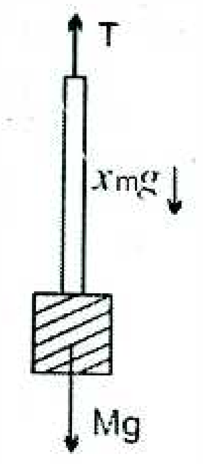
(vi) A wave is generated at the lower end O in the string as shown.



The velocity at position x is given by

$$v = \frac{dx}{dt} = \sqrt{\frac{T}{m}} = \sqrt{\frac{Mg + xmg}{m}}$$

(a) When mL<<Mg



In this case Mg + xmg ≈ Mg

Then speed does not depends upon x, we get $t = \frac{L}{V} = \frac{L}{\sqrt{\frac{M}{m}}}$ □ $t = L\sqrt{\frac{m}{M}}$

(b) When mL not negligible.

In this case, speed depends upon x. Here

$\frac{d}{d} = \sqrt{\frac{M + x}{m}}$ □ Accl^n (a) $= \frac{d}{d}\sqrt{\frac{M + x}{m}}$

$= \frac{d}{d}\sqrt{\frac{M}{m} + gx} \times \frac{d}{d}$

$= \frac{d\left(\frac{M}{m}+g\right)^{1/2}}{d\left(\frac{M}{m}+g\right)} \times \frac{d}{d}\left(\frac{M}{m} + gx\right) \times \sqrt{\frac{M}{m} + gx}$

$= \frac{1}{2}\frac{1}{\sqrt{\frac{M}{m}+g}} \times g \times \sqrt{\frac{M}{m} + gx} = \frac{g}{2}$

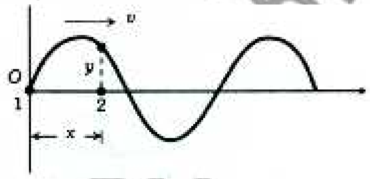
This pulse moves with acceleration g/2 up ward.

□ $L = \frac{1}{2}\left(\frac{g}{2}\right)t^2 + \sqrt{\frac{M}{m}}t$ □ $t = \sqrt{\frac{M}{m}}\left(\sqrt{1 + \frac{m}{M}} - 1\right)$

PROGRESSIVE WAVE. A periodic disturbance in a medium or space is called progressive wave. In progressive wave energy is transferred from one place to another by the vibration of the medium.

EQUATION OF PLANE PROGRESSIVE WAVE.

Suppose a plane simple harmonic wave travels from the origin along the positive direction of x-axis from left to right as shown in figure.

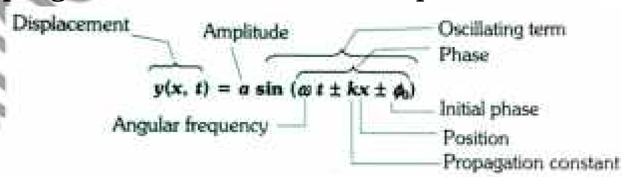


The displacement y of a particle 1 at 0 from its mean position at any time t is given by y (0, t) = a sinωt.

The wave reaches the particle 2 after time t = x/v. Hence displacement y of of a Particle 2 is given by

$y(x, t) = a \sin\omega\left(t - \frac{x}{v}\right) = a\sin(\omega \;\; - k \;\;)$ $[k = \frac{\omega}{v}]$

The general equation of plane progressive wave with initial phase is



Various forms of progressive wave function.

(i) $y = a \;\; (\omega t - kx)$

(ii) $y = a \;\; \left(\omega t - \frac{2}{\lambda}x\right)$

(iii) $y = a \;\; \left(t - \frac{x}{\lambda}\right)$

(iv) $y = a \;\; \frac{2}{T}\left(t - \frac{T}{\lambda}x\right)$

(v) $y = a \;\; \frac{2}{\lambda}(vt - x)$

(vi) $y = a. \;\; \left(t - \frac{x}{v}\right)$

PARTICLE VELOCITY:

We have $y = a \;\; \frac{2\pi}{\lambda}(vt - x)$......(i)

Differentiating with respect to t

$\frac{d}{d} = a\cos\frac{2\pi}{\lambda}(vt - x).\left(\frac{2\pi}{\lambda}v\right)$.........(ii)

$$\boxed{\text{Particle velocity}(V) = \frac{d}{d} = a\omega\cos\frac{2\pi}{\lambda}(vt - x)}$$

Again differentiating with respect to x.

$\frac{d}{d} = a\cos\frac{2\pi}{\lambda}(vt - x).\left(-\frac{2\pi}{\lambda}\right)$ ....(iii)

Dividing (ii) by (iii), we get

$\frac{d}{d} \Big/ \frac{d}{d} = -v$ □ $\boxed{\frac{d}{d} = -v\frac{d}{d}}$

Which is the relation between particle and wave velocity.

i.e., velocity of particle = - v × slope of the displacement.

Using equation (ii) we get,

$$V = a\left(\frac{2\pi}{\lambda}v\right)\sqrt{1 - \sin^2\frac{2\pi}{\lambda}(vt - x)}$$

$$= a\left(\frac{2\pi}{\lambda}v\right)\sqrt{1 - \frac{y^2}{a^2}} = \omega\sqrt{a^2 - y^2}$$

i.e., Maximum velocity at y = 0, V_m = aω

PHASE VELOCITY: v = nλ is called phase velocity of the wave.

NOTE : In a pure harmonic wave the phase velocity and the wave velocity are the same. In a group of velocity and the wave velocity are two separate entities.

PARTICLE ACCELERATION.

From particle acceleration

$$\frac{d}{d} = a\cos\frac{2\pi}{\lambda}(vt - x).\left(\frac{2\pi}{\lambda}v\right)..........(i)$$

Differentiating with respect to t

$$\frac{d^2y}{dt^2} = -a\sin\frac{2\pi}{\lambda}(vt - x).\left(\frac{2\pi}{\lambda}v\right).\left(\frac{2\pi}{\lambda}v\right)$$

$$\boxed{\text{Particle Accl}^n\ \frac{d^2y}{d^2} = -a\left(\frac{2\pi}{\lambda}v\right)^2\sin\frac{2\pi}{\lambda}(vt - x)}...(ii)$$

Differentiating (i) with respect to x

$$\frac{d^2y}{d^2} = -a\left(\frac{2\pi}{\lambda}x\right)^2\sin\frac{2\pi}{\lambda}(vt - x) \quad ....(iii)$$

Dividing equation (ii) by (iii) we get

$$\frac{d^2y/d^2}{d^2y/d^2} = v^2 \quad □ \quad \frac{d^2y}{d^2} = v^2\frac{d^2y}{d^2}$$

i.e., Acceleration of particle =

v^2 × the curvature of the displacement curve.

$$\boxed{\frac{d^2y}{d^2} = \frac{1}{v^2}\frac{d^2y}{d^2}}$$

which is called differential equation of wave.

Using equation (ii) we get

$$\frac{d^2y}{d^2} = -a(\omega)^2\sin\frac{2\pi}{\lambda}(vt - x) = \omega^2 y$$

i.e., Accl^n will be maximum at y = a

$f_m = -\omega^2 a$

IMPORTANT RELATIONS FOR NUMERICAL SOLVING

(i) Angular frequency ω = co-efficient of t

(ii)Propagation constant k =co-efficient of x

Wave speed $v = \frac{c \quad o\ t}{c \quad o\ x} = \frac{\omega}{k}$

(iii) wave length $\lambda = \frac{2}{c \quad o\ x}$

(iv) Time period $T = \frac{2}{c \quad o\ t}$

(v) Frequency $n = \frac{co\ o\ t}{2}$

PHASE DIFFERENCE AND PATH DIFFERENCE. At any instant t, if $□_1$ & $□_2$ are the phases of two particles whose distances from the origin are x_1 and x_2 respectively then $□_1 = (\omega t - kx_1)$ and $□_2 = (\omega t - kx_2)$ □ $□_1 - □_2 = k(x_1 - x_2)$

□ $\boxed{P \quad d \quad (\Delta□) = \frac{2}{\lambda} \times P \quad d \quad (\Delta x)}$

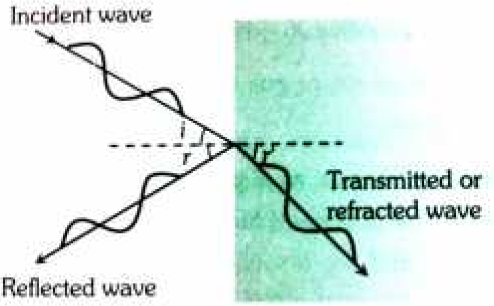
PHASE DIFFERENCE AND TIME DIFFERENCE.

$\boxed{P \quad d \quad (\Delta□) = \frac{2}{T} \times P \quad d \quad (\Delta t)}$

RARER AND MEDIUM: A medium is said to be denser (relative to the other) if the speed of wave in this medium is less than the speed of wave in other medium.

In comparison to air speed of sound is more in water; hence water is rarer medium for sound wave w.r.t. air.

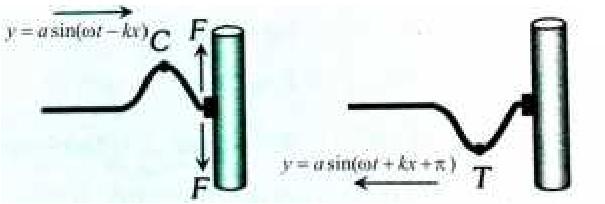
In reflection and refraction frequency remains same.



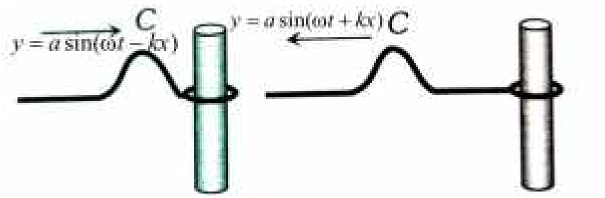
In case of refraction or transmission,

$$\frac{s}{s \quad r'} = \frac{v_i}{v_t}$$

REFLECTION OF WAVE. When the incident wave reaches a fixed end, it exert an upward pull on the end, according to Newton's third law the fixed end exerts an equal and opposite downward force on the string. It results as inverted pulse or phase change of π. Crest (C) reflects as trough (T) and vice- versa. Time change by $\frac{T}{2}$ and path changes by $\frac{\lambda}{2}$.



When a wave is reflected from a free end,



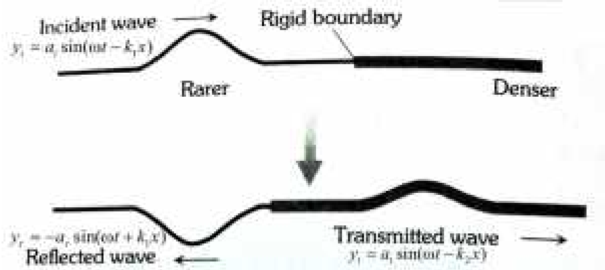
Then there is no change of phase (as there is no reaction force).

Crest (C) reflects as crest (C) and trough (T) reflects as trough (T), Time changes by zero and path changes by zero.

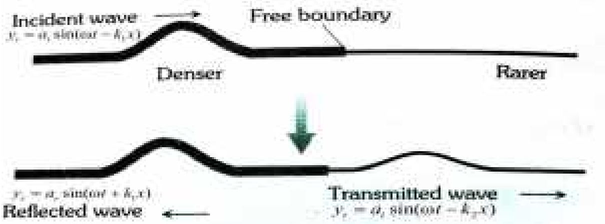
Exception : Longitudinal pressure waves suffer no change in phase from rigid end. i.e., compression pulse reflects as compression pulse. On other hand if longitudinal pressure wave reflects from free end, it suffer a phase change of π, i.e., compression reflects as refraction and vice-versa.

WAVE IN A COMBINATION OF STRING

(a) Wave goes from thin to thick string



(b) Wave goes from thick to thin string.



Ratio of amplitudes:

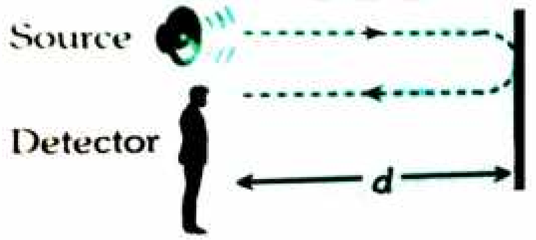
$\frac{a_r}{a_i} = \frac{k_1-k_2}{k_1+k_2} = \frac{v_2-v_1}{v_2+v_1}$ and

$\frac{a_t}{a_i} = \frac{2k_1}{k_1+k_2} = \frac{2v_2}{v_2+v_1}$

ECHO.

An eco is simply the repetition of speaker's own voice caused by reflection at a distant surface.

If there is a sound reflector at a distance d from source, then the time interval between original sound and it's echo at the site of source will be

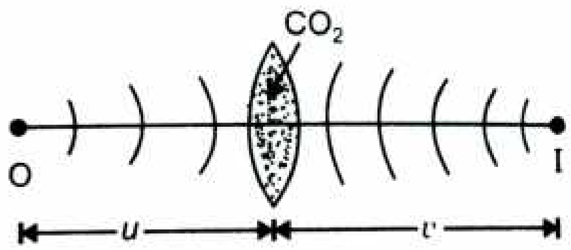


$t = \frac{d}{v} + \frac{d}{v} = \frac{2}{v}$ As the persistence of hearing for human ear is o.1 sec, therefore in order that an echo of short sound will be heard if t > 0.1□ $\frac{2}{v}$ > o.1 □ $d > \frac{v}{2}$

If v = speed of sound = 340 m/s

then d > 17m

Sound Lens .A large convex lens made of plastic sheets filled with a denser gas CO_2, such lenses are used to concentrate sound wave at a particular point.



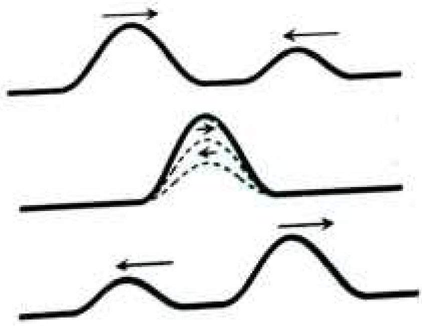
Sound lens

A watch may be placed as an object at O; and on gradually moving to the other side, a point I is obtained where the ticking of the watch is loudest. This is the image I. Knowing the distance u of the object, and of the image, v from the lens, the focal length of this sound lens is obtained as

$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ □ $f = \frac{u}{u+v}$

SUPERPOSITION OF WAVE

The displacement at any time due to a number of waves meeting simultaneously at a point in a medium is the vector sum of the individual displacements due to each one of the waves at that point at the same time.



If $\vec{y_1}, \vec{y_2}, \vec{y_3}$ ……… are the displacement at a particular position, due to individual waves, then the resultant displacement.
$\vec{y} = \vec{y_1} + \vec{y_2} + \vec{y_3}$ …… This principle holds good as long as the amplitude of wave wire is not too large.

APLICATION OF SUPERPOSITION PRICIPLE.

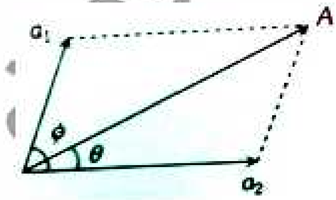
1. Interference of waves (Adding waves that differ in phase)
When two waves of same frequency, same wavelength, and same velocity are superimpose to each other then it called interference of wave.
In interference energy is neither created nor destroyed but is redistributed.
If $y_1 = a_1 \sin\omega t$, $y_2 = a_2 \sin(\omega t + \phi)$ then by principle of superposition

$Y = a_1 \sin\omega t + a_2 \sin(\omega t + \phi)$
$= a_1 \sin\omega t + a_2 \{\sin\omega t \cos\phi + \cos\omega t \sin\phi\}$
$= \sin\omega t\ (a_1 + a_2\cos\phi) + (a_2 \sin\phi)\cos\omega t$
$= \sin\omega t \cdot A\cos\theta + A\sin\theta \cdot \cos\omega t$

$$Y = A\sin(\omega t + \theta)$$

Where $A\cos\theta = (a_1 + a_2\cos\phi)$
$A\sin\theta = (a_2\sin\phi)$

Hence $A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$



And $\tan\theta = \frac{a_2 \sin\phi}{a_1 + a_2\cos\phi}$

Intensity (I) ∝ (Amplitude)^2 ⇒ $\frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2$

Therefore resultant intensity is given by

$$I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$$

If $I_1 = I_2 = I_0$ (say) then $I = 4I_0\cos^2\frac{\phi}{2}$

⇒ $\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2 = \left(\frac{a_1+a_2}{a_1-a_2}\right)^2$

⇒ $I_a = a_1^2 + a_2^2$

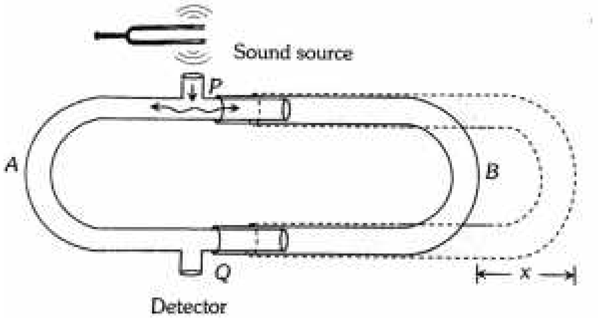
APLICATION OF INTERFERENCE OF SOUND (Stethoscope)

Stethoscope is a medical instrument used frequently by doctors for making a rough diagnosis of the diseases in that part of the body which are either inaccessible or accessible only through major operation. It consists of a cup-shaped metal piece having a diaphragm encased in it. Two rubber tubes of equal lengths are attached to the metal piece. Sound waves picked up by the diaphragm from the heart or lungs are in phase when they reach the ear through the rubber tubes and due to constructive interference, a loud sound is heard.

CONSTRUCTIVE & DESTRUCTIVE INTERFERENCE.

CONSTRUCTIVE INTERFERENCE	**DESTRUCTIVE INTERFERENCE**
When the wave meet a point with same phase, constructive interference is obtained at that point (i.e., maximum sound)	When the waves meet a point with opposite phase, destructive interference is obtained at that point (i.e., minimum sound)
Path difference between the waves at the point of observation $\Delta = n\lambda$ (i.e., even multiple of $\lambda/2$)	Path difference $\Delta = (2n-1)\frac{\lambda}{2}$ (i.e., odd multiple of $\lambda/2$)
Phase (ϕ) = 0^0 or $2n\pi$	Phase (ϕ) = 180^0 or $(2n-1)\pi$
Resultant Amplitude $A_{max} = a_1 + a_2$	Resultant Amplitude $A_{min} = a_1 - a_2$

QUINK'S TUBE. This is an apparatus used to demonstrate the phenomenon of interference and also used to measure velocity of sound in air. This is made up of two U-tube A and B as shown in figure. Here the tube B can slid in and out from the tube A. There are two opening P and Q in the tube A. At opening P, a tuning fork or a sound source of known frequency n_0 Is placed and the other opening a detector is placed to detect the resultant sound of interference occurred due to superposition of two sound waves coming from the tube A and B.



Initially tube B is adjusted so that detector detects a maximum. At this instant if length of path covered by the two waves from P and Q from the side of A and side of B are l_1 and l_2 respectively then for constructive interference we must have

$l_2 - l_1 = N\lambda$ .... (1)

If now tube B is further pulled out by a distance x so that next maximum is obtained and the length of path from the side of B is l'_2 then we have

$l'_2 = l_2 + 2x$ .... (2)

Where x is the displacement of the tube. For next constructive interference of sound at point Q, we have

$l'_2 - l_1 = (N+1)\lambda$ .... (3)

From equation (1), (2) and (3), we get

$l'_2 - l_2 = 2x = \lambda \quad \therefore x = \frac{\lambda}{2}$

Thus by experiment we get wavelength of sound as for two successive point of constructive interference, the path difference must be λ. As the tube B is pulled out by x, this introduced a path difference 2x in the path of sound wave through tube B. If the frequency of the source is known, n_0, the velocity of sound in the air filled in tube can be gives as

$v = n_0.\lambda = 2n_0x$.

BEATS (Adding waves that differ in frequency).

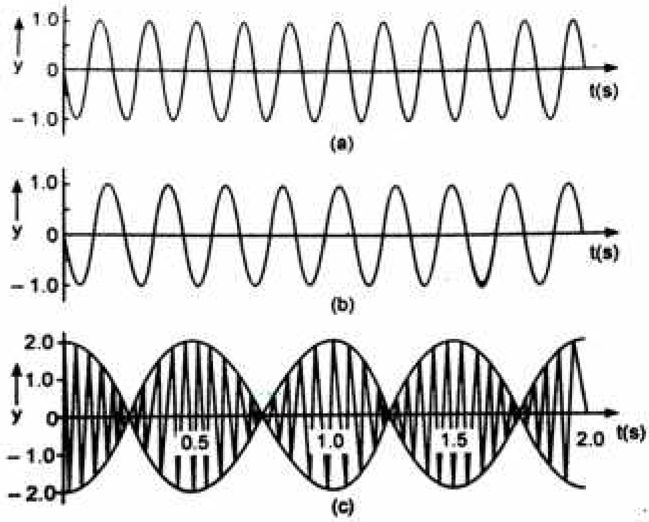
The phenomenon of alternate rise and fall of sound at regular intervals is called beats.

Formation of beats and frequency of beats:

Beats is a result of superposition of two waves of nearly same frequencies. If two waves of frequency ν_1 and ν_2, superimpose on one another then number of beats formed per unit time i.e., the frequency of beats is given by

Frequency of beats $\nu = \nu_1 - \nu_2$

Graphical representation of beats: The graphical formation of beats has been shown in following fig. Plot (a) represents one superposing wave of frequency ν_1= 11Hz and plot (b) represents another wave of frequency ν_2 = 9Hz. The plot (c) shows the effect of superposition i.e., beats formation. The number of beats formed is 2 beats per second.



Analytical treatment of beats:

Let $y_1 = a \sin\omega_1 t$ & $y_2 = a \sin\omega_2 t$

Applying the principle of superposition, we get

$$y = a \sin\omega_1 t + a \sin\omega_2 t$$

$$= 2a \sin\left(\frac{\omega_1+\omega_2}{2}\right)t . \cos\left(\frac{\omega_1-\omega_2}{2}\right)t$$

$$= \left[2a\cos\left(\frac{\omega_1-\omega_2}{2}\right)t\right]. \sin\left(\frac{\omega_1+\omega_2}{2}\right)t$$

$$\boxed{y = A\sin\omega t}$$

Where $A = 2a\cos\left(\frac{\omega_1-\omega_2}{2}\right)t$

$\omega_1 = 2\pi\nu_1$ & $\omega_2 = 2\pi\nu_2$

$$\omega = \left(\frac{\omega_1+\omega_2}{2}\right), \qquad \omega = 2\pi\nu$$

$$\boxed{y = A\sin 2\pi\nu t}$$

<u>Condition for maximum sound</u>:

Intensity of sound will be maximum when

$$\cos\left(\frac{\omega_1-\omega_2}{2}\right)t = \cos\frac{2\ (\nu_1-\nu_2)t}{2} = \pm 1 = \cos 2n\pi$$

where n = o, 1, 2, 3, ...

$(\nu_1 - \nu_2)t = n$ □ $t = \frac{n}{\nu_1-\nu_2}$

So, the time interval between two successive maxima is $\frac{1}{\nu_1-\nu_2}$

<u>Condition for minimum sound</u>:

$$\cos\left(\frac{\omega_1-\omega_2}{2}\right)t = \cos\frac{2\ (\nu_1-\nu_2)t}{2} = 0 = \cos(2n-1)\pi/2$$

Where n = 1, 2, 3, ...

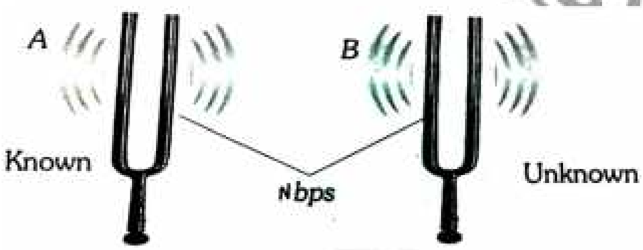
$(\nu_1 - \nu_2)t = \frac{2n-1}{2}$ □ $t = \frac{2n-1}{2(\nu_1-\nu_2)}$

So, the time interval between two successive minima is $\frac{1}{\nu_1-\nu_2}$

USE OF BEATS

(a) To determine unknown frequency

Suppose a tuning fork of known frequency (n_A) is sounded together with another tuning fork of unknown frequency (n_B) and N beats heard per second.



BY LOADING	
If B is loaded with Wax	**If A is loaded with Wax**
If N increases $n_B = n_A - N$	If N increases $n_B = n_A + N$
If N decreases $n_B = n_A + N$	If N decreases $n_B = n_A - N$
If N remains same $n_B = n_A + N$	If N remains same $n_B = n_A - N$
If N becomes zero $n_B = n_A + N$	If N becomes zero $n_B = n_A - N$
BY FILING	
If B is filed	**If A filed**
If N increases $n_B = n_A + N$	If N increases $n_B = n_A - N$
If N decreases $n_B = n_A - N$	If N decreases $n_B = n_A + N$
If N remains same $n_B = n_A - N$	If N remains same $n_B = n_A + N$
If N becomes zero $n_B = n_A - N$	If N becomes zero $n_B = n_A + N$

(b) use in mines

The presence of dangerous gases in mines may be detected by the use of beats. The apparatus consists of two identical pipes; one blown from the air from a reservoir and other from the air in the mines. If two 'airs' are the same, then the two pipes will give the notes of the same frequency. But if the air in the mine is polluted with dangerous gases, these lighter gases would be having a greater velocity of sound waves and that pipe will give a note of higher frequency. This will produced beats. Thus, the method serves as an early warning system to safeguard workers against possible dangerous explosions.

STATIONARY WAVE (Adding waves that differ in direction)

Whenever two progressive waves of the same wavelength and amplitude travel with the same speed through a medium in opposite direction and superimpose to each other, they give rise to waves called stationary or standing wave.

Stationary waves are of two types:

(i) Transverse stationary wave. These are formed due to the superposition of two progressive transverse waves.

Example. The stationary waves produced in the vibrating string of a sonometer.

(ii)Longitudinal stationary waves. These are formed due to the superposition of two progressive longitudinal waves.

Example. The stationary waves formed in the vibrating air columns in the pipes.

STANDING WAVES IN STRING.

Let a progressive wave pulse moving –ve x-axis we get, $y_1 = a\sin(\omega t + kx)$

After reflection, the wave pulse travels in +ve x-axis it is given by $y_2 = -a\sin(\omega t - kx)$

On superposition of two wave pulses, the displacement of the resultant wave pulse will be $y = y_1 + y_2$

$y = 2\cos\frac{(\omega t + k + \omega t - k)}{2}\sin\frac{(\omega t + k - \omega t + k)}{2}$

$= (2a \sin kx) \cos\omega t$

$$\boxed{y = \left(2a \sin\frac{2}{\lambda}x\right)\cos\frac{2}{\lambda}vt}$$

$$\boxed{y = A\cos\frac{2}{\lambda}vt} \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

Where $A = \left(2a\sin\frac{2}{\lambda}x\right)$= Resultant amplitude

Which is the equation of stationary wave.

POSITION OF NODE.

$2a\sin\frac{2}{\lambda}x = 0 = \sin o^{0}, \sin\pi, \sin 2\pi \ldots\ldots \sin n\pi$

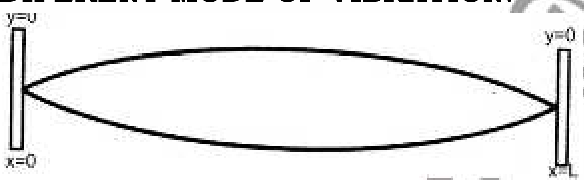
$$\boxed{x = n\frac{\lambda}{2} = 0, \frac{\lambda}{2}, \lambda, \frac{3}{2}, \ldots\ldots\ldots\ldots}$$

POSITION OF ANTINODE.

$2a\sin\frac{2}{\lambda}x = \pm 1 = \sin\frac{\pi}{2}, \sin\frac{3}{2}, \sin\frac{5}{2} \ldots\ldots \sin\frac{(2-1)\pi}{2}$

$$\boxed{x = (2n-1)\frac{\lambda}{4} = \frac{\lambda}{4}, \frac{3}{4}, \frac{5}{4}, \ldots\ldots}$$

DIFERENT MODE OF VIBRATION.



At first boundary, y = 0 at x = 0 for all t
Equation (1) satisfy the second boundary condition, If A = 0 at x = L for all t

$2a\sin\frac{2}{\lambda}L = 0 = \sin o^{0}, \sin\pi, \sin 2\pi \ldots\ldots \sin n\pi$

$$\boxed{L = n\frac{\lambda}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Here n = 1, 2, 3, ... Correspond to the first, second and third mode of vibration.

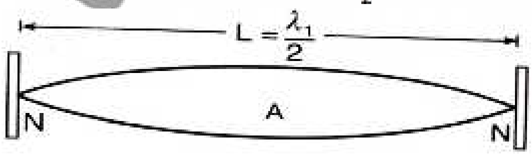
FIRST MODE OF VIBRATION.

(FUNDAMENTAL FREQUENCY)

Let n = 1,

wave length of stationary wave is λ_1

From equation (2) $\quad L = \frac{\lambda_1}{2} \quad$ or $\lambda_1 = 2L$



$f_1 = \frac{v}{\lambda_1} = \frac{1}{\lambda_1}\sqrt{\frac{T}{m}} = \frac{1}{2}\sqrt{\frac{T}{m}}$ $\quad \boxed{f_1 = \frac{1}{2}\sqrt{\frac{T}{m}}}$

SECOND MODE OF VIBRATION.

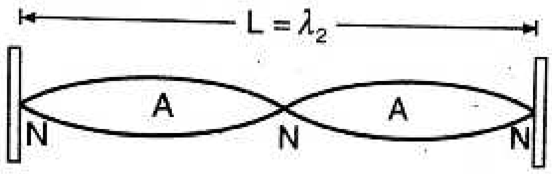
(First over tone)

Let n = 2,

wave length of stationary wave is λ_2

From equation (2) $\quad L = 2\frac{\lambda_2}{2} \quad$ or $\lambda_2 = L$

$f_2 = \frac{v}{\lambda_2} = \frac{1}{\lambda_2}\sqrt{\frac{T}{m}} = \frac{1}{L}\sqrt{\frac{T}{m}} = \frac{2}{2}\sqrt{\frac{T}{m}} = 2f_1$



THIRD MODE OF VIBRATION.

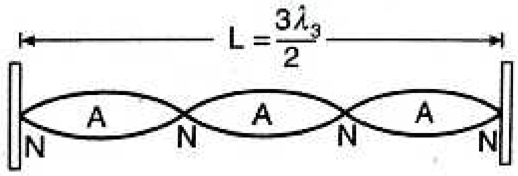
(Second over tone)

Let n = 3,

wave length of stationary wave is λ_3

From equation (2) $\quad L = 3\frac{\lambda_3}{2} \quad$ or $\lambda_3 = \frac{2}{3}$

$f_3 = \frac{v}{\lambda_3} = \frac{1}{\lambda_3}\sqrt{\frac{T}{m}} = \frac{3}{2}\sqrt{\frac{T}{m}} = 3f_1$

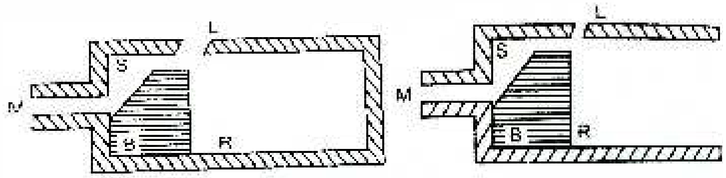


i.e., $f_1, f_2, f_3 \ldots\ldots\ldots\ldots = 1, 2, 3 \ldots\ldots$

It give odd and even harmonic both.

ORGAN PIPE.

It is a wind instrument in which sound is produced by setting into vibrations an air column in it. It consists of a wooden or metallic hollow or closed tube called resonator (R).



Closed organ pipe $\qquad$ Open organ pipe

A narrow tapering opening called mouth-piece (M) is provided at one end of the resonator as shown in figure. A slanting called bevel (B) is fitted near the mouth piece. The height of the bevel is such that there is only a narrow slit S between the bevel and the wall of the resonator. A sharp edge (L) is provided in the wall of the resonator. This is called lip of the pipe.

STATIONARY WAVE IN CLOSED ORGAN PIPE.

Let a progressive wave pulse moving –ve x-axis we get, $y_1 = a\sin(\omega t + kx)$

After reflection, the wave pulse travels in +ve x-axis it is given by $y_2 = -a\sin(\omega t - kx)$

On superposition of two wave pulses, the displacement of the resultant wave pulse will be $y = y_1 + y_2$

$$y = 2\cos\frac{(\omega t + kx + \omega t - kx)}{2}\sin\frac{(\omega t + kx - \omega t + kx)}{2}$$

$$= (2a\sin kx)\cos\omega t$$

$$\boxed{y = \left(2a\sin\frac{2}{\lambda}x\right)\cos\frac{2}{\lambda}vt}$$

$$\boxed{y = A\cos\frac{2}{\lambda}vt} \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

Where $A = \left(2a\sin\frac{2}{\lambda}x\right)$ = Resultant amplitude

This is the equation of stationary wave.

POSITION OF NODE.

$$2a\sin\frac{2}{\lambda}x = 0 = \sin 0^{0}, \sin\pi, \sin 2\pi \ldots\ldots \sin n\pi$$

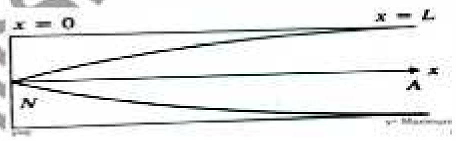
$$\boxed{x = n\frac{\lambda}{2} = 0, \frac{\lambda}{2}, \lambda, \frac{3}{2}, \ldots\ldots\ldots\ldots}$$

POSITION OF ANTINODE.

$$2a\sin\frac{2}{\lambda}x = \pm 1 = \sin\frac{\pi}{2}, \sin\frac{3}{2}, \sin\frac{5}{2} \ldots\ldots \sin\frac{(2\ -1)\pi}{2}$$

$$\boxed{x = (2n-1)\frac{\lambda}{4} = \frac{\lambda}{4}, \frac{3}{4}, \frac{5}{4}, \ldots\ldots}$$

MODE OF VIBRATION IN CLOSED ORGAN PIPE.



At first boundary y = 0, x = 0 for all t, equation (1) satisfy the second boundary condition if A = max^{m} at x = L for all t

$$2a\sin\frac{2}{\lambda}L = \pm 1 = \sin\frac{\pi}{2}, \sin\frac{3}{2}, \sin\frac{5}{2} \ldots\ldots \sin\frac{(2\ -1)\pi}{2}$$

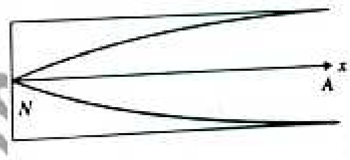
$$\boxed{L = (2n-1)\frac{\lambda}{4}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Here n = 1, 2, 3, ……. Correspond to the first, second and third mode of vibration

FIRST MODE OF VIBRATION.

(FUNDAMENTAL FREQUENCY)

Let λ_1 is wavelength of the stationary waves set up corresponding to n = 1, then from the equation (2), we have $L = \frac{\lambda_1}{4}$



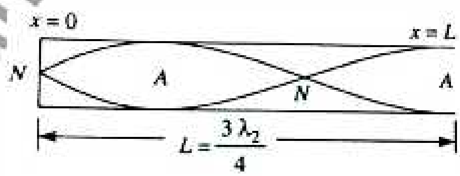
but $f_1 = \frac{v}{\lambda_1} = \frac{v}{4}$

It is called fundamental frequency or pitch of the tone or the first harmonic.

SECOND MODE OF VIBRATION.

(FIRST OVER TONE)

Let λ_2 is wavelength of the stationary waves set up corresponding to n = 2, then from the equation (2), we have $L = \frac{3\ }{4}$



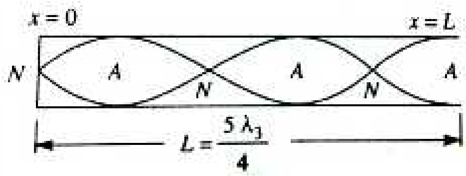
but $f_2 = \frac{v}{\lambda_2} = 3\frac{v}{4} = 3f_1$

It is called third harmonic.

THIRD MODE OF VIBRATION.

(SECOND OVER TONE)

Let λ_3 is wavelength of the stationary waves set up corresponding to n = 3, then from the equation (2), we have $L = \frac{5\ }{4}$



but $f_3 = \frac{v}{\lambda_3} = 5\frac{v}{4} = 3f_1$

It is called fifth harmonic.

i.e., $f_1, f_2, f_3 \ldots\ldots\ldots\ldots = 1, 3, 5 \ldots\ldots$

It give odd harmonic only.

OPEN ORGAN PIPE.

Let a progressive wave pulse moving +ve x-axis we get, $y_1 = a\sin(\omega t - kx)$

After reflection, the wave pulse travels in

-ve x-axis it is given by $y_2 = a\sin(\omega t + kx)$
On superposition of two wave pulses, the displacement of the resultant wave pulse will be $y = y_1 + y_2$

$$y = 2\sin\frac{(\omega t - kx + \omega t + kx)}{2}\cos\frac{(\omega t - kx - \omega t - kx)}{2}$$

$$= (2a\cos kx)\sin\omega t$$

$$\boxed{y = \left(2a\cos\frac{2}{\lambda}x\right)\sin\frac{2}{\lambda}vt}$$

$$\boxed{y = A\sin\frac{2}{\lambda}vt} \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

Where $A = \left(2a\cos\frac{2}{\lambda}x\right)$ = Resultant amplitude

This is the equation of stationary wave.

POSITION OF NODE.

$$2a\cos\frac{2}{\lambda}x = 0 = \cos\frac{\pi}{2}, \cos\frac{3\pi}{2}, \cos\frac{5\pi}{2} \ldots\ldots \cos\frac{(2n-1)\pi}{2}$$

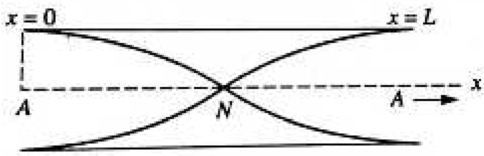
$$\boxed{x = (2n-1)\frac{\lambda}{4} = \frac{\lambda}{4}, \frac{3}{4}, \frac{5}{4}, \ldots\ldots}$$

POSITION OF ANTINODE.

$$2a\cos\frac{2}{\lambda}x = \pm 1 = \cos 0, \cos\pi, \cos 2\pi, \ldots\ldots \cos n\pi$$

$$\boxed{x = n\frac{\lambda}{2} = \frac{\lambda}{2}, \lambda, \frac{3}{2}, \ldots\ldots}$$

MODE OF VIBRATION IN OPEN ORGAN PIPE.



At first boundary y = max^m, at x = 0 for all t, equation (1) satisfy the second boundary condition if y = max^m , at x = L for all t

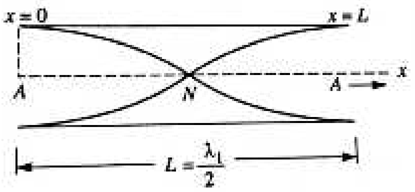
$$\cos\frac{2}{\lambda}L = \pm 1 = \cos 0, \cos\pi, \cos 2\pi \ldots\ldots\ldots \cos n\pi$$

$$\boxed{L = n\frac{\lambda}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

FIRST MODE OF VIBRATION.

(FUNDAMENTAL FREQUENCY)

Let λ_1 is wavelength of the stationary waves set up corresponding to n = 1, then from the equation (2), we have $L = \frac{\lambda_1}{2}$

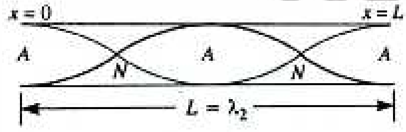


but $f_1 = \frac{v}{\lambda_1} = \frac{2}{L}$ It is called fundamental frequency or pitch of the tone or the first harmonic.

SECOND MODE OF VIBRATION.

(FIRST OVER TONE)

Let λ_2 is wavelength of the stationary waves set up corresponding to n = 2, then from the equation (2), we have $L = \frac{2\,\lambda_2}{2}$



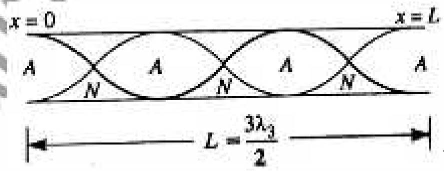
but $f_2 = \frac{v}{\lambda_2} = \frac{v}{L} = \frac{2}{2} = 2f_1$

It is called second harmonic.

THIRD MODE OF VIBRATION.

(SECOND OVER TONE)

Let λ_3 is wavelength of the stationary waves set up corresponding to n = 3, then from the equation (2), we have $L = \frac{3\,\lambda_3}{2}$



but

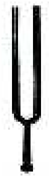
$$f_3 = \frac{v}{\lambda_3} = 3\frac{v}{2} = 3f_1$$

It is called third harmonic.

i.e., $f_1, f_2, f_3 \ldots\ldots\ldots = 1, 2, 3 \ldots\ldots$

It give odd and even harmonic both.

TUNING FORK . A **tuning fork** is an acoustic resonator in the form of a two-pronged fork with the prongs (tines) formed from a U-shaped bar of elastic metal (usually steel). It resonates at a specific constant pitch when set vibrating by striking it against a surface or with an object, and emits a pure musical tone after waiting a moment to allow some high overtones to die out. The pitch that a particular tuning fork generates depends on the length and mass of the two prongs. It is frequently used as a standard of pitch to tune musical instruments.



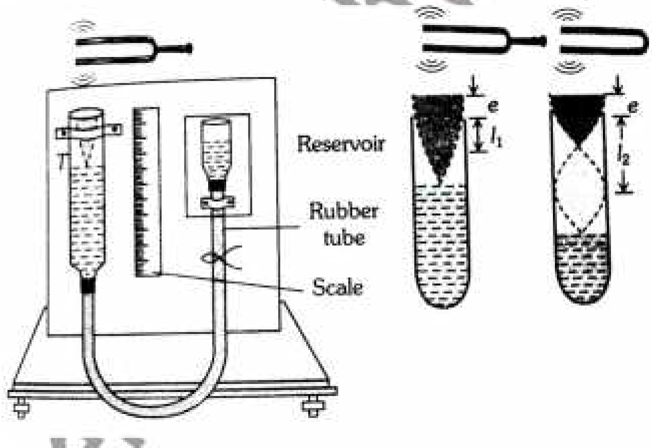
The frequency of a tuning fork depends on its dimensions and the material from which it is made:

$$f = \frac{1.8^{\ \ 2}}{2\ \ l^2}\sqrt{\frac{E}{\mu A}}$$

Where:

- f is the frequency the fork vibrates at in hertz.
- l is the length of the prongs in metres.
- E is the Young's modulus of the material the fork is made from in Pascal's.
- I is the second moment of area of the cross-section in metes to the fourth power.
- ρ is the density of the material the fork is made from in kilogram's per cubic meter.
- A is the cross-sectional area of the prongs (tines) in square meters

RESONANCE TUBE.
It is used to determine velocity of sound in air by the help of a tuning fork of known frequency.
It is a closed organ pipe having an air column of variable length. When a tuning



fork is brought over it's mouth. It's air column vibrates with the frequency of the fork. If the length of the air column is varied until it's natural frequency equals the frequency of the fork, then the column resonant and emits a loud note.
If l_1 and l_2 are lengths of first and second resonances, then we have $l_1 + e = \frac{\lambda}{4}$

and $l_2 + e = 3\frac{\lambda}{4}$ □$l_2 - l_1 = \frac{\lambda}{2}$ □$\lambda = 2(l_2 - l_1)$
Speed of sound at room temperature
$v = n\lambda = 2n(l_2 - l_1)$
Also $\frac{l_2+e}{l_1+e} = 3$ □ $l_2 = 3l_1 + 2e$ i.e., second resonance is obtained at length more than thrice the length of first resonance.

DOPPLER EFFECT. The change in the frequency of sound because of relative motion between the source and the observer of sound is called Doppler's effect. This gives information regarding the change in frequency only. It does not say about intensity of sound.
Apparent frequency (f').
Source emits sound at a certain frequency. It is called the actual frequency (f) and the observer receives this sound is called apparent frequency (f'), it is given by

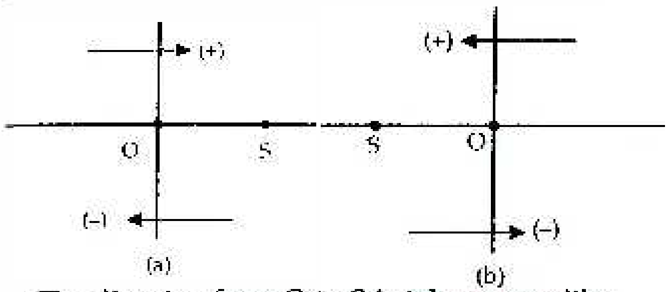
$$f' = \frac{v'(V \quad a \ s \quad r \quad ti \ o \quad)}{\lambda'(W \quad a \ w \quad r \quad ol \quad)} \quad .. (1)$$

□ $v' = v \pm v_o$ &

$$\text{Wavelength } (\lambda') = \frac{V}{F}\frac{(v \pm v_s)}{(f)}$$

Using eqⁿ (1) we get, $\boxed{f' = \frac{(v \pm v_o)}{(v \pm v_s)} f}$ ……… (2)

This is such formula which describes Doppler effect for all possible situation.



The direction from O to S is taken as positive and the direction from S to O is taken as negative whatever be the situation.

(a) Source in motion and observer at rest.
(i) When source moving toward stationary observer. In this case $v_s \to -ve$, $v_o \to 0$

using equation (2), we get $f' = \frac{v}{v-v_S} f$

i.e., $f' > f$

(ii) When source moving away from stationary observer.

In this case $v_S \to +ve, v_O \to 0$

using equation (2), we get $f' = \frac{v}{v+v_S} f$

i.e., $f' < f$

(b) observer in motion and source at rest.

(i) When observer moving toward stationary source.

In this case $v_S \to 0, v_O \to +ve$

using equation (2), we get $f' = \frac{v+v_O}{v} f$

i.e., $f' > f$

(ii) When observer moving away from stationary source.

In this case $v_S \to 0, v_O \to -ve$

using equation (2), we get $f' = \frac{v-v_O}{v} f$

i.e., $f' < f$

(c) Source and observer both in motion.

(i)If the source and observer are approaching each other.

In this case $v_S \to -ve, v_O \to +ve$

using equation (2), we get $f' = \frac{v+v_O}{v-v_S} f$

i.e., $f' > f$

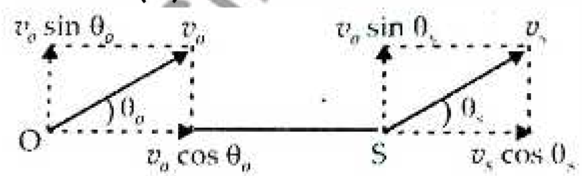
(ii) If the source and observer are moving away from one another.

In this case $v_S \to +ve, v_O \to -ve$

using equation (2), we get $f' = \frac{v-v_O}{v+v_S} f$

i.e., $f' < f$

NOTE. The effect of angle between the velocities of the source (S) and observer (O).



$$f' = \left(\frac{v+v_O \cos\theta_O}{v+v_S \cos\theta_S}\right) f$$

Where θ_O = Angle which the velocity of observer in making with the direction $\overrightarrow{OS}$

θ_S = Angle which the velocity of source is making with the direction $\overrightarrow{OS}$.

CONDITION WHEN DOPPLER EFFECT IS NOT APPLIED.

(i) When source (S) and observer (O) both at rest.

(ii) When medium alone is moving.

(iii) When S and O move in such a way that distance between S and O remains constant.

(iv) When source S and observer O, are moving in mutually perpendicular directions.

(v) If the velocity of source and observer is equal to or greater than the sound velocity then Doppler effect is not seen.