ELASTICITY

1. **Elasticity** is the property of a solid by virtue of which it regains its original shape or size after the externally applied deforming force has been removed. Quartz is a nearly perfectly elastic material.

2. **Plasticity** is opposite of elasticity. If a body does not regain its original shape/size on the removal of the external stresses, howsoever small they may be, it is called a *plastic body*. Putty and paraffin wax are nearly perfectly plastic substances.

3. **Explanation of Elasticity on the basis of Atomic model.** The atoms of a solid are held together in a regular array by electric forces. The solid remains in its natural equilibrium state under these forces. If the solid is compressed by an external force, the distance between the atoms decreases resulting in a net force of repulsion between the atoms. When the external force is removed, the interatomic force pushes the atoms back to their initial positions so that the solid returns to its original size. Similarly, a restroring force is also developed when the solid is stretched.

4. **Stress.** Restoring force per unit area is called *stress*.

   Magnitude of the stress = F/A

   The SI unit of stress is N-m^2 or pascal (Pa) and its dimensional formula is $[ML^{-1}T^{-2}]$.

5. **Types of stress**

   (i) **Normal stress.** When a deforming force acts normally over a surface of a body, then *the internal restoring force set up per unit area of the body is called normal stress.*

   Normal stress can further be as

   (a) **Tensile stress.** If there is an increase in the dimensions of the body in the direction of force applied, the stress set up is called tensile stress.

   (b) **Compression stress.** If there is a decrease in the dimensions of the body due to force applied, the stress set up is called compression stress.

   (ii) **Tangential stress.** When a deforming force, acting tangentially to the surface of a body changes the shape of the body, then the stress set up in the body is called tangential stress.

   (iii) **Hydrostatic stress.** If a body is subjected to a uniform and equal pressure from all sides, it is under hydrostatic stress.

6. **Strain.** Strain is the deformation per unit of the relevant dimension of the substance.

   Since the strain is a ratio of change in dimension to original dimension, it has no units and is a dimensionless quantity.

7. **Type of strain.** Since the change in configuration involves a change either in length, volume or shape of the body, hence the strain is of three types.

   (i) **Longitudinal strain.**

   $$\text{Longitudinal strain} = \frac{\text{change in length } (\Delta l)}{\text{original length } (l)}$$

   (ii) **Volumetric strain.**

   $$\text{Volumetric strain} = \frac{\text{change in volume } (\Delta V)}{\text{original volume } (V)}$$

   (iii) **Shearing strain.** If the deforming force produces a change in the shape of the body without changing its volume, the strain thus produced is called shearing strain. It is defined as angle in radian through which a plane perpen-dicular to the fixed surface of the body gets turned under the effect of tangential force.

   The shearing strain is also defined as the ratio of displacement of a surface under a tangential force to the perpendicular distance of the displaced surface from the fixed surface.

8. *Hooke's Law states that*

   within proportionality limit,

   stress $\propto$ strain

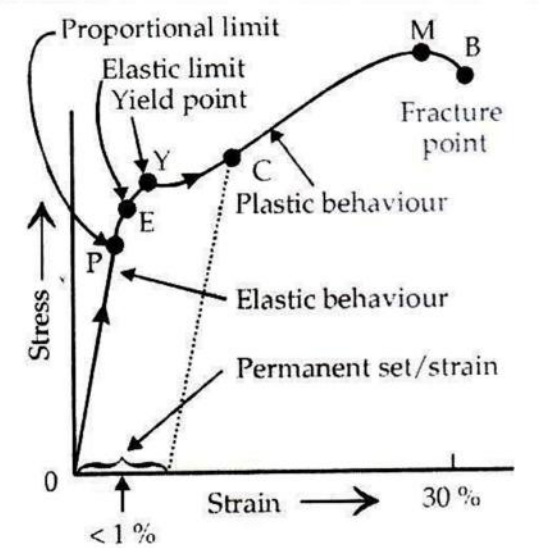
   Or stress = k × strain

   The proportionality constant k is called the *modulus of elasticity*. It is denoted as:

   1. **Young's Modulus (Y)** in case of longitudinal stresses.
   2. **Shear Modulus or Modulus of Rigidity (η or G)** in case of tangential stresses.

3. **Bulk Modulus (K)** in case of volumetric stresses.

9. **Stress-strain diagram**



A typical stress-strain curve for a ductile metal

10. **Elastic and plastic bodies.** According to the property of elasticity, the bodies may be divided into two groups:

    Elastic Bodies. The bodies which regain completely their original state after removal of deforming forces are called perfectly elastic.

    Plastic Bodies. The bodies which do not return to their original state after removal of deforming forces but remain deformed permanently are called perfectly plastic.

    In practice, no body shows perfectly elastic or perfectly plastic behavior but all bodies behave between two limits. But we may consider quartz as nearly perfectly elastic and wax or wet clay nearly perfectly plastic.

11. **Proportional limit (Point P).** In a certain range of the stress-strain curve for many structural materials, the stress and strain are proportional to each other, so that any increase in stress will result in a proportionate increase in strain. The stress at the limit of proportionality point P is known as the proportional limit.

12. **Elastic limit / Limit of Elasticity (Point E).** *The limiting deforming force below which a body retains its property of elasticity and above which it loses its property of elasticity is called the limit of elasticity.*

13. **Yield point (Point Y).** The point on the stress-strain curve beyond the elastic limit whereafter the material continues to deform without an increase in load is known as the yield point. This phenomenon occurs only in certain ductile materials.

14. **Yield strength / stress.** The stress at yield point Y is called yield strength/stress.

15. **Ultimate strength (Point M).** The stress beyond which the material keeps on flowing even if the external load is decreased is known as ultimate strength. The ultimate strength or the tensile strength is therefore the maximum stress developed at M by the material based on the original cross-sectional area. A brittle material breaks when stressed to the ultimate strength, whereas a ductile material will continue to stretch.

16. **Breaking strength (Point B).** The value of the stress at which localized deformation or necking occurs in the specimen, and the load falls off is known as breaking strength. The breaking strength is determined by dividing the breaking load by the original cross-sectional area, is always less than the ultimate strength. For a brittle material, the ultimate strength and breaking strength coincide.

17. **Ductility.** If there is large plastic deformation in between yield point and breaking point, the metal is said to be ductile. Under such circumstances, rods of metals can be drawn into wires.

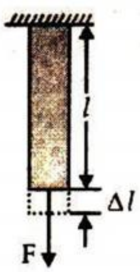
18. **Brittleness.** If the wire breaks into pieces on reaching beyond elastic limit, it is called brittle.

19. **Malleability.** When a solid is compressed, a stage is reached beyond which it can not recover its original shape even after removal of deforming force. This is elastic limit or yield point for compression. *The yield point obtained during compression is called crushing point.* After this, metals are said is be malleable *i.e.* they can be hammered or rolled into sheets.

20. Elastomers. These are the materials for which stress-strain graph is not a straight line even within the elastic limit, and the strain produced is in much larger proportion than the stress. Such materials have no plastic region, the breaking point lies just close to the elastic limit. Rubber is an example of elastomers. In our body, the elastic tissue of aorta (the large vessel carrying blood from the heart) is an elastomer.

21. Types of modulus of elasticity. Corresponding to three types of strain, there are three types of modulus of elasticity.

22. Tension and compression young's modulus (Y)



$Y = \frac{\sigma}{\varepsilon}$ or $\boxed{Y = \frac{F/A}{\Delta l/l_0}}$

The unit of Young's modulus is the same as that of stress, *i.e.*, $N\text{-}m^{-2}$ or pascal (Pa).

23. Work done in stretching a wire. Work done or total energy stored in the elongated wire,

$$W = \text{Avg. force} \times \text{displacement}$$

$$= \left(\frac{0+F}{2}\right) \times (\Delta l)$$

$$\boxed{W = \frac{1}{2} \times F \times \Delta l}$$

$$W = \frac{1}{2} \times \left(\frac{F}{A} \times A\right) \times \left(\frac{\Delta l}{l_0} \times l_0\right)$$

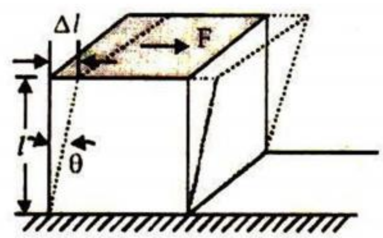
$$= \frac{1}{2} \times \frac{F}{A} \times \frac{\Delta l}{l_0} \times (A \times l_0)$$

$$= \frac{1}{2} \text{ stress} \times \text{strain} \times \text{volume of specimen}$$

$$\therefore \boxed{W = \frac{1}{2} \times \sigma \varepsilon V}$$

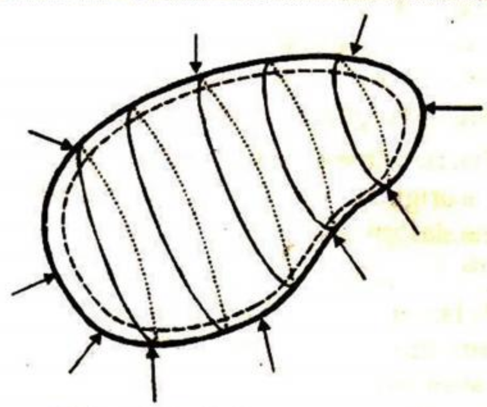
$$\therefore \boxed{\text{Energy stored per unit volume of the specimen} = \frac{1}{2} \text{stress} \times \text{strain}}$$

24. Shearing, Modulus of rigidity (G or η)



$$\boxed{G = \frac{F/A}{\Delta l/l}}$$

25. Hydraulic stresses, Bulk modulus (K or B)



Volumetric strain

$$K = \frac{\text{stress}}{\text{strain}} = \frac{\Delta p}{-\Delta V/V}$$

$$\boxed{K = -\frac{\Delta p \cdot V}{\Delta V}}$$

26. Compressibility (*C*). Reciprocal of Bulk Modulus is called **compressibility (C).**

$$\therefore \boxed{C = \frac{1}{K}}$$

27. Points to remember

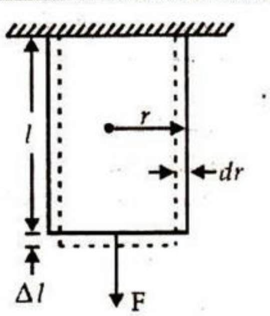
1. The modulus of elasticity has units and dimensional formula as of stress or pressure.

2. Higher the value of modulus of elasticity, more elastic is the material.
3. Young's modulus of elasticity-Y and modulus of rigidity η exist only for solids because liquids and gases cannot be deformed in one dimensional only and they also cannot undergo shear strain. The bulk modulus of elasticity K exists for all the three states of matter–solid, liquid and gas.
4. The solids are highest in elasticity and gases are least elastic because for the given stress applied, the gases are more compressible than that of solids.
5. Young's modulus of the material of a wire is numerically equal to the stress.
6. Quartz is the nearest approach to a perfectly elastic body.
7. The compressibility of gases varies with pressure and temperature.

28. Elastic fatigue is characteristic of an elastic body because of which its behaviour becomes less elastic under the action of repeated alternating deforming stresses.

29. Elastic after effect. The delay in regaining the original shape or size after the removal of the deforming stresses is called elastic after effect.

30. Poisson's Ratio (σ). Whenever a rod is subjected to longitudinal stress, its length increases but its diameter decreases. Decrease in radius per unit radius is called lateral strain.



Poisson's ratio is defined as

$$= -\frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$\sigma = -\frac{\Delta r/r}{\Delta l/l_0} \text{ or } \sigma = \left|\frac{\Delta r/r}{\Delta l/l_0}\right|$$

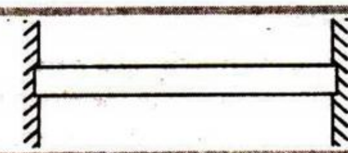
The minus sign is introduced in the definition of Poisson's ratio to make it a positive quantity. The value of σ is always less than 1/2. For most solids, it lies between 1/4 and 1/3.

31. Factors affecting elasticity of a material

1. **Hammering and rolling** result in a decrease in the plasticity of the material because of breaking-up of crystal grains into smaller units. Thus elasticity of the material increases.
2. **Annealing** increases the elasticity by removal of internal stresses.
3. **Effect of the presence of impurities** can be both ways i.e. it can increase or decrease the elasticity of the material. The type of effect depends upon the nature of the impurity present in the material.
4. **Effect of temperature.** In most cases, an increase in temperature of the material causes decrease in elasticity of the material.

32. Thermal stresses

I. If the two ends of a rod are fixed and its temperature is increased.



$$L_t = L_0(1 + \alpha t) \text{ or } L_{t_2} = L_{t_1}[1+\alpha(t_2-t_1)]$$

where L_0 = original length at 0°C.
L_t = length at raised temperature
α = temperature coefficient of linear expansion.
t = rise in temperature.

$$\therefore \Delta L = L_t - L_0 = \alpha L_0 t \quad \text{... (i)}$$

Thus the rod is under compression and its ΔL is given in equation (i). Let us do back calculations to find out the stress (s) which would have resulted by this ΔL.

$$Y = \frac{s}{\Delta L/L_0}$$

$$\text{or } \Delta L = \frac{sL_0}{Y} \quad \text{... (ii)}$$

Putting (ii) in (i) we get

$$\alpha \cdot L_0 t = \frac{sL_0}{Y}$$

or $\boxed{s = \alpha Y t}$

If F is the force developed in wire, then

F = Stress × cross-sectional area = (Y α t).A

$\Rightarrow F = Y A \alpha t$

Clearly, the force or tension developed in the wire is independent of length of wire and is directly proportional to a temperature-change. If cooled sufficiently, the wire may break.

II. Similarly, the stresses which will set up in the gas which is enclosed within fixed walls, when its temperature is raised by t.

$$\boxed{\Delta p = K\gamma t}$$

where Δp = Increase in pressure of the gas.

K = Bulk Modulus

γ = Coefficient of cubical expansion of the gas.

t = rise in temperature.

33. Effect of compression on density (ρ) of a liquid

$$\rho = \frac{m}{V}$$

Take log on both sides

$$\log \rho = \log m - \log V$$

Differentiating we get

$$\frac{\Delta\rho}{\rho} = 0 - \frac{\Delta V}{V} = -\frac{\Delta V}{V} \quad \text{... (i)}$$

Δm will be zero, because mass remains constant.

Bulk Modulus (K) is given by

$$K = -\frac{\Delta p \cdot V}{\Delta V}$$

$$\Rightarrow \quad -\frac{\Delta V}{V} = \frac{\Delta p}{K} \quad \text{... (ii)}$$

Put (ii) in (i)

$$\frac{\Delta\rho}{\rho} = \frac{\Delta p}{K}$$

i.e. $$\frac{\rho' - \rho}{\rho} = \frac{\Delta p}{K}$$

$$\Rightarrow \quad \boxed{\rho' = \rho\left(1 + \frac{\Delta p}{K}\right)}$$

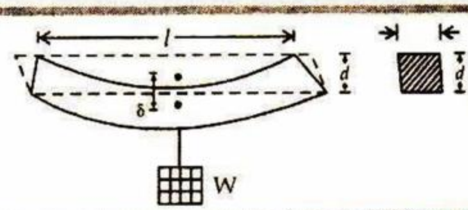
or $$\boxed{\rho' = \rho(1 + C \cdot \Delta p)}$$

where

C = Compressibility which has already been defined as the reciprocal of Bulk Modulus (K) of the liquid.

34. Applications of Elastic Behaviour of Materials

(i) While designing a building, the structural design of columns, beams and supports.



When a load W is attached at middle point of a bar, the depression δ produced at middle point in the bar is given by

$$\delta = \frac{Wl^3}{4bd^3Y}$$

(ii) Use of wire-ropes in cranes.

(iii) Design of bridges.

(iv) The answer of the question why the maximum height of a mountain on earth is nearly 10 km is also provided by considering the elastic properties of rocks.

(v) Hollow shaft is found to be stronger than a solid shaft made of equal volume of same material.

(vi) The metallic parts of the machinery are never subjected to a stress beyond elastic limit.

1. ***What is elasticity ?***

✔ The tendency of a body to regain its original state—length, volume and shape on the removal of the deforming forces is called elasticity, e.g., quartz fibre, phosphor bronze possess elasticity.

2. ***What is an elastomer?***

✔ It is a substance which can be elastically stretched to large values of strain.

3. ***Which of the two forces-deforming or restoring is responsible for elastic behaviour or a substance ?***

✔ Restoring force.

4. ***What are the factors on which modulus of elasticity of a material depends ?***

✔ Nature of the material and the manner in which it is deformed.

5. ***What is origin of stress?***

✔ Deforming force displaces atoms from their actual positions. These displaced atoms exert an opposing force, which appears as stress.

6. ***Name the factors which affect the property of elasticity of a solid.***

✔ (i) Presence of impurities. (ii) Change of temperature. (iii) Effect of hammering, rolling and annealing.

7. ***What is elastic limit ?***

✔ The maximum stress upto which the body can regain its original state on the removal of the deforming force is called elastic limit.

8. ***What is breaking stress?***

✔ Breaking stress is the ratio of maximum load to which the wire is subjected divided by the original cross-sectional area.

9. ***Why strain has no unit?***

✔ Strain is the ratio of two similar quantities, due to which it is dimensionless and it has no unit.

10. ***A heavy wire is suspended from a roof but no weight is attached to its lower end. Is it under stress ? Justify your answer.***

✔ A heavy wire (even when no weight is attached to it) is under stress, when it is suspended from a roof. It is because, the weight of the heavy wire acts as the deforming force.

11. ***Define Young's modulus of elasticity.***

✔ Refer to point no. 8 of *To The Point* Theory.

12. ***A wire increases by 10^{-3} of its length, when a stress of 10^{8} N m^{-2} is applied on it. What is the Young's modulus of the material of the wire ?***

✔ Here, stress = 10^8 N m^{-2} ; strain = 10^{-3}

$$\therefore\ Y = \frac{\text{stress}}{\text{strain}} = \frac{10^8}{10^{-3}} = 10^{11}\ \text{N m}^{-2}$$

13. ***If only the diameter of a wire is doubled, how will the following parameters be affected?***
(a) Extension for same load?
(b) Load for same extension?

✔ We have, $Y = \frac{MgL}{Al} = \frac{4MgL}{\pi D^2 l}$

(a) For same 'Mg' $D^2 l$ is constant. Doubling of D make D^2 four times, hence 'l' becomes one-fourth.

(b) For same 'l', Mg/D^2 is constant. Four time D^2 will make Mg four times.

14. ***If only the length of a wire is doubled, how will the following parameters be affected?***

(a) Extension for same load?

(b) Load for same extension?

✔ We have, $Y = MgL/Al$,

(a) For same Mg, L/l is constant. When L is doubled, l also becomes double.

(b) For same l, MgL is constant. When L is doubled, Mg becomes half.

15. ***Under what condition, the restoring forces are equal and opposite to the external deforming force ?***

✔ When the body is deformed within its elastic limit.

16. ***Mention one situation where the restoring force is not equal and opposite to the applied force.***

✔ The restoring force is not equal and opposite to the applied force when the body is deformed beyond elastic limit.

17. ***What is Rigidity modulus ?***

✔ The ratio between tangential stress and shear strain is called rigidity modulus $\eta = \frac{F/A}{\theta}$.

18. ***Why does a liquid have zero modulus of rigidity?***

✔ Because liquids do not have shape of itself.

19. ***Which of the three types of elasticity (Y, K and η) is possessed by all the three states of matter (solid, liquid and gas) ?***

✔ The volume elasticity (K) is possessed by all the three states of the matter.

20. ***What is the value of Y for a perfectly rigid body?***

✔ Infinite.

21. ***What is the shear modulus of a liquid?***

✔ Zero.

22. ***What is the bulk modulus of a perfectly rigid body?***

✔ Infinite.

23. ***A wire is stretched to double its length. What is the value of longitudinal strain?***

✔ Unity.

24. ***What is elastic fatigue ?***

✔ Loss of the elastic behaviour by a material due to the repeated application of alternating strains is called elastic fatigue.

25. ***What do you understand by elastic after-effect ?***

✔ If the body recovers to original status slowly after the removal of the deforming force then this is called elastic after-effect.

26. ***Why do springs become slack after a prolonged use?***

✔ The elasticity of the material decreases due to repeated deformation in the long run. The elastic limit also decreases and springs become deformed permanently.

27. ***A hard wire is broken by bending it repeatedly in opposite direction. Why ?***

✔ It is because of the loss of the strength of the material due to repeated alternating strains, to which the wire is subjected.

28. ***In stretching a wire, work has to be performed. Why ?***

✔ When a wire is stretched, interatomic forces come into play and these forces oppose the increase in length of the wire. Therefore, in order to stretch the wire, work has to be done against the interatomic forces.

29. ***What is the work done in stretching a wire ?***

✔ Work done in stretching a wire is

$$\frac{1}{2}\text{ Stress} \times \text{Strain} \times \text{Volume or } \frac{1}{2}\text{ Y(Strain)}^2 \times \text{Volume or } \frac{1}{2}\frac{(\text{Stress})^2}{\text{Y}} \times \text{Volume}$$

30. ***When a wire is stretched, work has to be done. What happens to the work done during the stretching of the wire ?***

✔ The work done in stretching the wire is stored in it in the form of the elastic potential energy.

31. ***What will happen to the potential energy if a wire is (a) compressed (b) stretched?***

✔ In both the cases, work has to be done on the wire. So, the potential energy will increase in both the cases.

32. ***Why do we prefer steel to copper in the manufacture of spring.***

✔ It is because Young's modulus of steel is large as compared to that of copper.

33. ***Why are electric poles given hollow structure ?***

✔ A hollow shaft is stronger than a solid shaft made from the same and equal amounts of material.

34. ***What is Poisson's ratio?***

✔ It is the ratio of lateral strain and linear strain.

Short Answer Questions

1. ***Explain atomic theory of elasticity?***

A. In a body, atoms are at a definite distance from each other called equilibrium distance. At this distance, the nearer force of repulsion is balanced by the farther force of attraction.

On stretching the body, the distance between atoms increases. Here forces of attraction come into play. These take the atoms back to their initial positions after the removal of the stretching forces. On compressing the body, the distance between atoms decreases. Then forces of repulsion come into play. Atoms return back to their initial positions, after the removal of the compressing forces.

2. ***What is meant by plasticity ?***

A. The inability associated with a body in regaining the original status on the removal of the deforming forces is called plasticity, e.g., bakelite, putty are plastic in behaviour.

3. ***Why is a spring made of steel and not of copper ?***

A. A better spring will be one, in which a large restoring force is developed on being deformed. This, in turn, depends upon the elasticity of the material of the spring. As Young's modulus of steel is greater than that of copper, steel is preferred to manufacture a spring.

4. ***Two wires of same length and material but of different radii are suspended from a rigid support. Both carry the same load. Will the stress, strain and extension in them be same or different?***

A. Stress $= \frac{F}{a} = \frac{F}{\pi r^2}$

Since the two wires carry the same load (F) but are of different radii, the stress in the two wires will be different.

Now, extension, $l = \frac{F \times L}{a \times Y} = \frac{F \times L}{\pi r^2 \times Y}$

Since for the two wires, F, L and Y are same but r is different, the extension in two wires will be *different*.

Also, strain, $\frac{l}{L} = \frac{F}{a \times Y} = \frac{F}{\pi r^2 \times Y}$

Obviously, strain will also be different in the two wires.

5. ***The length of a metallic wire is L_1, when the tension in the wire is T_1; and is L_2, when the tension is T_2. Find the original length of the wire.***

A. Let L and a be the original length and the area of cross-section of the wire. If on applying a force F, extension produced is l, then

$$Y = \frac{F/a}{l/L} = \frac{FL}{al}$$

In the first case, $F = T_1$ and $l = L_1 - L$

$$\therefore \quad Y = \frac{T_1 L}{a(L_1 - L)}$$

In the second case, $F = T_2$ and $l = L_2 - L$

$$\therefore \quad Y = \frac{T_2 L}{a(L_2 - L)}$$

From the equations (i) and (ii), we have $\frac{T_1 L}{a(L_1 - L)} = \frac{T_2 L}{a(L_2 - L)}$

or $T_1(L_2 - L) = T_2(L_1 - L)$ or $L(T_2 - T_1) = T_2 L_1 - T_1 L_2$ or $L = \frac{T_2 L_1 - T_1 L_2}{T_2 - T_1}$

6. ***A cable is cut to half of its original length. Why this change has no effect on the maximum load, the cable can support ?***

A. The breaking stress is a constant for the given material. We know that
breaking load = breaking stress × area of cross-section
When the cable is cut to half of its length, the area of cross-section does not change. Hence, there is no effect on the maximum load (breaking load), the cable can support.

7. ***The breaking force for a wire is F. What will be the breaking force for (a) two parallel wires of the same size (b) for a single wire of double the thickness ?***

A. (a) When two wires of the same size are suspended in parallel ; a force equal to 2 F has to be applied on the parallel combination, so that a force F equal to the breaking force for the wire acts on each of the two wires.

(b) $F = \frac{YAl}{L} = \frac{Y(\pi d^2)l}{4L}$ or $F \propto d^2$

If the wire is of double the thickness *i.e.* of double the diameter, then breaking force will be 4 F.

8. *Why is a solid ductile or brittle ? [Explain on the basis of stress versus strain graph]*

A. In the stress versus strain graph for a wire, the portion of graph between elastic limit and breaking point is called plastic region.

A solid is said to be more ductile, if the plastic region of the graph is longer ; and brittle, if the plastic region is shorter.

9. *Which is more elastic, steel or rubber? Explain.*

A. Steel is more elastic, than rubber.

Let wires of steel and rubber of same length and cross-section are loaded with same load. Let l_s and l_r be increase in their lengths.

Then, for steel $Y_s = MgL/Al_s$

and for rubber $Y_r = MgL/Al_r$

Dividing $\frac{Y_s}{Y_r} = \frac{l_r}{l_s}$

since $l_r > l_s$, we get $Y_s > Y_r$. Thus steel having greater value of elastic constant is more elastic.

10. *Identical springs of steel and copper are equally stretched. In which case more work is done?*

A. Steel is more elastic than copper. So, for producing a given extension, more force is required in the case of steel spring as compared to copper spring. Thus, more work is required to be done on the steel spring as compared to copper spring.

11. *A wire of length L and cross-sectional area A is made of a material of Young's modulus Y. If the wire is stretched by an amount x, the work done is ______. Fill in the blank.*

$$F = \frac{YAx}{L}$$

$$W = \int_0^x dW = \int_0^x \frac{YAx}{L} dx = \frac{YA}{L}\int_0^x x\,dx = \frac{YA}{L}\left[\frac{x^2}{2}\right]_0^x = \frac{YAx^2}{2L}$$

12. *A wire of cross-sectional area A and Young's modulus of elasticity Y is clamped rigidly between two supports. What tension will be developed in the wire when its temperature is increased through it? Given : α = coefficient of linear expansion.*

A. $\alpha = \frac{\Delta l}{lt}$ or $\Delta l = l\alpha t$

Longitudinal strain $= \frac{\Delta l}{l} = \alpha t;$

Stress = $Y\alpha t$

Tension F = $Y\alpha t$ × cross-sectional area = $YA\alpha t$

13. *Graphite consists of planes of carbon atoms. Such carbon atoms in the planes are held by very weak force. What kind of elastic properties do you expect from graphite ?*

Due to weak attractive forces between the carbon atoms, the atomic planes easily get laterally displaced w.r.t. each other, when a small tangential stress is applied. Since

$$\text{modulus of rigidity } (\eta) = \frac{\text{tangential stress}}{\text{shear strain}}$$

such a material possesses a small value of the modulus of rigidity.

14. ***Why do spring balances show wrong readings after they have been used for a long time ?***

A. When spring balance has been used for a long time, the spring in the balance gets fatigued and there is loss in elastic strength of the spring. Then, for a given load, the extension in the spring is much more and hence the spring balance gives wrong readings.

15. ***Why are the bridges declared unsafe after long use ?***

A. A bridge undergoes continuous alternating strains a large number of times everyday. Therefore, a long use of the bridge results in the loss of its elastic strength. In other words, after a long use, the strain produced is quite large for a given stress and may lead to the collapse of the bridge. To avoid this the bridges are declared unsafe after a long use.

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1. *A steel wire of length 4.7 m and cross-section 3.0×10^{-5} m^2 stretches by the same amount as a copper wire of length 3.5 m and cross-section 4.0×10^{-5} m^2 under a given load. What is the ratio of the Young's modulus of steel to that of copper ?*

Sol. Let suffix 1 refer to steel wire and 2 to copper wire.

Young's modulus of steel wire,

$$Y_1 = \frac{M_1 g L_1}{\pi r_1^2 l_1} = \frac{M_1 g L_1}{A_1 l_1} \quad ...(1)$$

and Young's modulus of copper wire,

$$Y_2 = \frac{M_2 g L_2}{\pi r_2^2 l_2} = \frac{M_2 g L_2}{A_2 l_2} \quad ...(2)$$

Dividing equation (1) and (2), we get

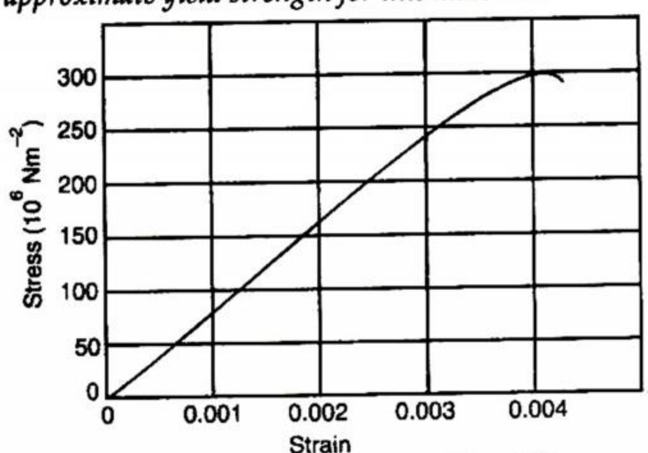
$$\frac{Y_1}{Y_2} = \frac{M_1 L_1}{A_1 l_1} \times \frac{A_2 l_2}{M_2 L_2} \quad ...(3)$$

Here, $M_1 = M_2$; $A_1 = 3.0 \times 10^{-5}$ m^2; $A_2 = 4.0 \times 10^{-5}$ m^2; $l_1 = l_2$; $L_1 = 4.7$ m and $L_2 = 3.5$ m.

Substituting the values in (3), we get

$$\frac{Y_1}{Y_2} = \frac{A_2 L_1}{A_1 L_2} = \frac{4.0 \times 10^{-5} \times 4.7}{3.0 \times 10^{-5} \times 3.5} = 1.79$$

2. *Figure shows the strain-stress curve for a given material. What are (a) Young's modulus and (b) approximate yield strength for this material?*



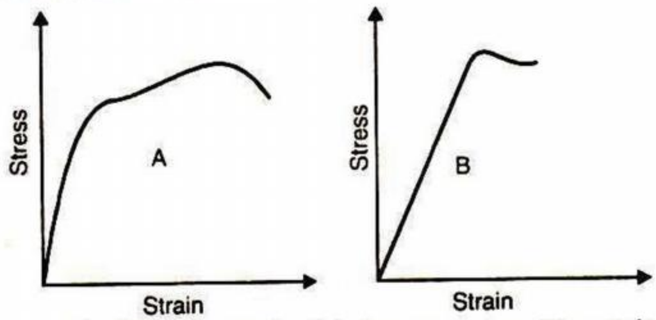
Sol. (*a*)

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{150 \times 10^6}{0.002} \text{ Nm}^{-2} = \frac{150 \times 10^6}{2 \times 10^{-3}} \text{ Nm}^{-2}$$

$$\boxed{Y = 7.5 \times 10^{10} \text{ Nm}^{-2}}$$

(*b*) Approximate yield strength will be equal to the maximum stress it can sustain without crossing the proportional limit. $\Rightarrow 300 \times 10^6$ Nm^{-2}

3. *The stress versus strain graphs for two materials A and B are shown in Figure (The graphs are to the same scale).*



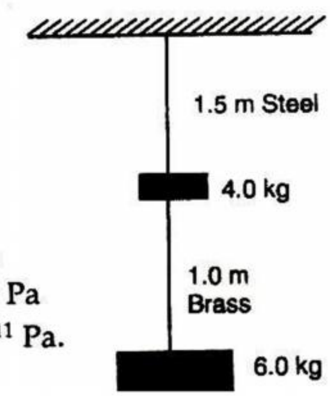
(a) *Which material has greater Young's modulus ?*

(b) *Which of the two is the stronger material ?*

Sol. (*a*) Material *A* has greater Young's modulus because Y = stress/strain *i.e.* steepness of the initial straight line portion.

(*b*) Material *A* is stronger because its Young's Modulus is greater.

4. *Two wires of diameter 0.25 cm, one made of steel and other made of brass are loaded as shown in Figure. The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m. Young's modulus of steel is 2.0×10^{11} Pa and that of brass is 0.91×10^{11} Pa. Compute the elongations of steel and brass wires.* $(1 Pa = 1 Nm^{-2})$



Sol. Given

$D_{steel} = 0.25$ cm

$L_{unloaded} = 1.5\ m = l_s$

$D_{Brass} = 0.25$ cm.

$L_{unloaded} = 1.0\ m = l_B$

$Y_{steeel} = 2.0 \times 10^{11}$ Pa

$Y_{brass} = 0.91 \times 10^{11}$ Pa.

For steel wire :

Net force on steel wire

$$F_S = 4 + 6 = 10 \text{ kgf} = 10 \times 9.8 \text{ N}$$

$$Y_{steel} = \frac{F_s \times l_s}{a_s \times \Delta l_s} \quad \left[a_s = \pi\left(\frac{D_{steel}}{2}\right)^2\right]$$

Putting the respective values,

$$\Delta l_s = \frac{10 \times 9.8 \times 1.5 \times 7}{22 \times (0.125 \times 10^{-2})^2 \times 2 \times 10^{11}}$$

$$\boxed{\Delta l_s = 1.49 \times 10^{-4} \text{ m}} \quad \textbf{Ans.}$$

For Brass wire :

Here $F_B = 6.0 \text{ kgf} = 6.0 \times 9.8$ N

$$Y_{Brass} = \frac{F_B \times l_B}{a_B \times \Delta l_B} \quad ...(ii)$$

$$\left[a_B = \pi\left(\frac{D_{Brass}}{2}\right)^2\right].$$

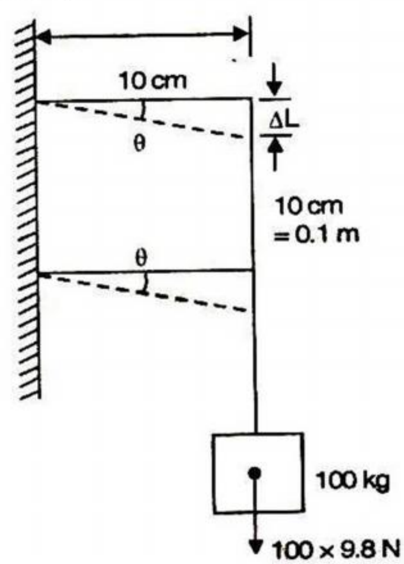
Putting the respective values in the above eqn. (*ii*)

$$\Delta l_B = \frac{(6 \times 9.8) \times (1.0 \times 7)}{22 \times (0.125 \times 10^{-2})^2 \times (0.91 \times 10^{11})}$$

$$\boxed{\Delta l_B = 1.3 \times 10^{-4} \text{ m}} \quad \textbf{Ans.}$$

5. *The edges of an aluminium cube are 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminium is 25 GPa. What is vertical deflection of this face ?* $(1 Pa = 1 Nm^{-2})$

Sol.



$$G = \frac{F/A}{\Delta L/L} = \frac{FL}{A.\Delta L}$$

$$\therefore \quad \Delta L = \frac{FL}{GA} = \frac{100 \times 9.8 \times 0.1}{25 \times 10^9 \times (0.1 \times 0.1)} \text{ m}$$

$$= 3.92 \times 10^{-7} \text{ m}$$

$$\approx 4 \times 10^{-7} \text{ m} \quad \textbf{Ans.}$$

6. *Four identical hollow cylindrical columns of steel support a big structure of mass 50,000 kg. The inner and outer radii of each column are 30 and 60 cm respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column. The Young's modulus of steel is 2.0×10^{11} Pa(1 Pa = 1 Nm^{-2}).*

Sol. Given $m = 50{,}000$ kg.

$$r_i = 30 \text{ cm} = 30 \times 10^{-2} \text{ m}$$

$$r_0 = 60 \text{ cm} = 60 \times 10^{-2} \text{ m}$$

$$\text{Force} = mg \text{ N} = 50{,}000 \times 9.8 \text{ N}$$

$$\text{Force} = 4.9 \times 10^5 \text{ N}$$

Force on each hollow cylinder

$$F' = \frac{4.9 \times 10^5}{4} \text{ N}$$

$$F' = 1.225 \times 10^5 \text{ N}$$

Now, $\quad Y_{steel} = 2.0 \times 10^{11}$ Pa.

and $\quad F = 1.225 \times 10^5$ N

Then compressional strain will be

$$\text{C.S.} = \frac{F/A}{Y_{steel}}$$

$$\text{C.S} = \frac{1.225 \times 10^4}{\pi (r_o^2 - r_i^2) \times Y_{steel}}$$

$$\text{C.S.} = \frac{1.225 \times 10^4 \times 7}{22 \times \left[(60 \times 10^{-2})^2 - (30 \times 10^{-2})^2\right] \times 2.0 \times 10^{11}}$$

$$\boxed{\text{C.S.} = 7.21 \times 10^{-7}} \quad \textbf{Ans.}$$

7. *A piece of copper having a rectangular cross-section of 15.2 mm × 19.1 mm is pulled in tension with 44,500 N force, producing only elastic deformation. Calculate the resulting strain.*

Sol. $\quad A = 15.2 \times 19.1 \times 10^{-6} \text{m}^2$;

$$\text{Strain} = \frac{\text{stress}}{\text{modulus of elasticity}} = \frac{F/A}{\eta}$$

$$= \frac{F}{A\eta} = \frac{44500}{(15.2 \times 19.2 \times 10^{-6}) \times 42 \times 10^9}$$

$$= 3.65 \times 10^{-3}.$$

8. *A steel cable with a radius of 1.5 cm supports a chairlift at a ski area. If the maximum stress is not to exceed 10^8 N m^{-2}, what is the maximum load the cable can support?*

Sol. Maximum load = maximum stress × area of cross section

$$= 10^8 \pi r^2 = 10^8 \times (22/7) \times (1.5 \times 10^{-2})^2$$

$$= 7.07 \times 10^4 \text{ N}$$

9. *A rigid bar of mass 15 kg is supported symmetrically by three wires each 2.0 m long. Those at each end are of copper and the middle one is of iron. Determine the ratios of their diameters if each is to have the same tension.*

Sol. As each wire has same tension F, so each wire has same extension due to mass of rigid bar. As each wire is of same length, hence each wire has same strain. If D is the diameter of wire, then

$$Y = \frac{4F/\pi D^2}{\text{strain}} \text{ or } D^2 \propto 1/Y$$

$$\therefore \quad \frac{D_{Cu}}{D_{iron}} = \sqrt{\frac{Y_{iron}}{Y_{Cu}}} = \sqrt{\frac{190 \times 10^9}{110 \times 10^9}}$$

$$= \sqrt{\frac{19}{11}} = 1.31$$

10. *A 14.5 kg mass, fastened to the end of a steel wire of unstretched length 1.0 m, is whirled in a vertical circle with an angular velocity of 2 rev/s at the bottom of the circle. The cross-sectional area of the wire is 0.065 cm^2. Calculate the elongation of the wire when the mass is at the lowest point of its path.*

Sol. $\quad A = 0.065 \times 10^{-4} \text{ m}^2$

Total pulling force on mass, when it is at the lowest position of the vertical circle is

$$F = mg + mr\,\omega^2$$

$$= mg + mr\,4\,\pi^2 \nu^2$$

$$= 14.5 \times 9.8 + 14.5 \times 1 \times 4 \times (22/7)^2 \times 2^2$$

$$= 142.1 + 2291.6 = 2433.7 \text{ N}$$

$$Y = \frac{F}{A} \times \frac{l}{\Delta l}$$

$$\Delta l = \frac{Fl}{AY} = \frac{2433.7 \times 1}{(0.065 \times 10^{-4}) \times (2 \times 10^{11})}$$

$$= 1.87 \times 10^{-3} \text{ m} = 1.87 \text{ mm}$$

11. *Compute the bulk modulus of water from the following data: Initial volume = 100.0 litre, pressure increase = 100.0 atm (1 atm = 1.013×10^5 Pa), Final volume = 100.5 litre. Compare the bulk modulus of water with that of air (at constant temperature). Explain in simple terms why the ratio is so large. (1 Pa = 1 Nm^{-2}).*

Sol. $V_{Initial}$ = 100.0 litre

$P_{increased}$ = 100.0 atm

$= (100 \times 1.013 \times 10^5)$ Pa

V_{final} = 100.5 litre.

Bulk modulus, B = ?

We know that

$$\Rightarrow \quad \frac{\Delta V}{V} = \frac{\text{Pressure}}{B}$$

$$\Rightarrow \quad B = \frac{1.013 \times 10^7}{0.5/100} Pa$$

$$B = \frac{1.013}{0.5} \times 10^9 Pa$$

$$\boxed{B = 2.026 \times 10^9 \text{ Pa}}$$

Bulk modulus (B) of air happens to be 1.0×10^5 Pa. Hence, B for water is much higher than that of air. It is because water is a liquid, hence, almost incompressible, while air is a gas which means it is easily compressible.

12. *What is the density of water at a depth where pressure is 80.0 atm, given that its density at the surface is 1.03×10^3 kg m^{-3} ? (Compressibility of water is 45.8×10^{-11} Pa^{-1}; 1 Pa = 1 Nm^{-2}).*

Sol. Given P = 80.0 atm

Density of water at the surface,

$\rho = 1.03 \times 10^3$ kgm^{-3}

Compressibility of water = 45.8×10^{-11} Pa^{-1}

Let ρ' be the density of water at the bottom where P = 80.0 atm.

We know that

$$V = \frac{\text{Mass}}{\text{density}} = \frac{M}{\rho}$$

Let V' be the volume at the depth

$$\Rightarrow \quad V' = M/\rho'$$

Then $$\Delta V = V - V' = M\left(\frac{1}{\rho} - \frac{1}{\rho'}\right)$$

$\Rightarrow$ Volumetric strain,

$$\frac{\Delta V}{V} = M\left(\frac{1}{\rho} - \frac{1}{\rho'}\right) \times \frac{\rho}{M}$$

$$\frac{\Delta V}{V} = 1 - \frac{\rho}{\rho'}$$

$$\Rightarrow \quad \frac{\Delta V}{V} = 1 - \frac{1.03 \times 10^3}{\rho'} \quad ...(i)$$

Also, $$B = \frac{1}{\text{Compressibility}}$$

and $$B = \frac{pV}{\Delta V}$$

$$\Rightarrow \quad \frac{\Delta V}{V} = \frac{p}{B}$$

$$\frac{\Delta V}{V} = (80.0 \times 1.013 \times 10^5) \times 45.8 \times 10^{-11}$$

$$\frac{\Delta V}{V} = 3.712 \times 10^{-3} \quad ...(ii)$$

Putting value obtained in equation (*ii*) in (*i*)

$$\Rightarrow \quad 1 - \frac{1.03 \times 10^3}{\rho'}, \quad \rho' = \frac{1.03 \times 10^3}{1 - 3.712 \times 10^{-3}}$$

$$\boxed{\rho' = 1.034 \times 10^3 \text{ kgm}^{-3}}$$

13. *Compute the fractional change in volume of a glass slab, when subjected to a hydraulic pressure of 10 atm.*

Sol. Here, p = 10 atm

$= 10 \times 1.013 \times 10^5$ Pa;

K = 37×10^9 Nm^{-2}

$$\text{Volumetric strain} = \frac{\Delta V}{V} = \frac{p}{K}$$

$$= \frac{10 \times 1.013 \times 10^5}{37 \times 10^9} = 2.74 \times 10^{-5}$$

$$\therefore \text{ Fractional change in volume} = \frac{\Delta V}{V}$$

$$= 2.74 \times 10^{-5}$$

14. *Determine the volume contraction of a solid copper cube, 10 cm on an edge, when subjected to a hydraulic pressure of 7.0×10^6 Pa.*

Sol. Here, L = 10 cm = 0.10 m;

$p = 7 \times 10^6$ Pa;

K = 140 GPa = 140×10^9 Pa

As $$K = \frac{pV}{\Delta V} = \frac{pL^3}{\Delta V}$$

or $$\Delta V = \frac{pL^3}{K} = \frac{(7 \times 10^6) \times (0.10)^3}{140 \times 10^9}$$

$$= 5 \times 10^{-8} \text{ m}^3 = 5 \times 10^{-2} \text{ mm}^3.$$

15. *How much should the pressure on a litre of water be changed to compress it by 0.10%?*

Sol. Here, $V = 1 \text{ litre} = 10^{-3}\ m^3$

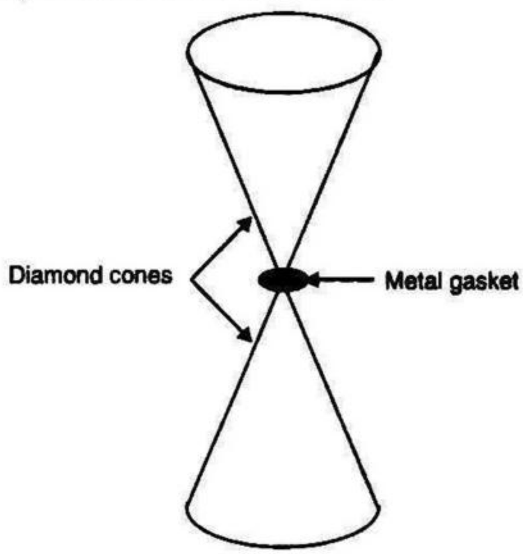
$$\Delta V/V = 0.10/100 = 10^{-3}$$

$$K = \frac{pV}{\Delta V}$$

or $p = K\frac{\Delta V}{V} = (2.2 \times 10^9) \times 10^{-3}$

$$= 2.2 \times 10^6 \text{ Pa}$$

16. *Anvils made of single crystals of diamond, with the shape as shown in Figure, are used to investigate behaviour of materials under very high pressures. Flat faces at the narrow end of the anvil have a diameter of 0.5 mm, and the wide ends are subjected to a compressional force of 50,000 N. What is the pressure at the tip of the anvil ?*



Sol. Given, $F = 50,000\text{ N}$;

$$d = 0.5 \text{ mm} = 5 \times 10^{-4} \text{ m}$$

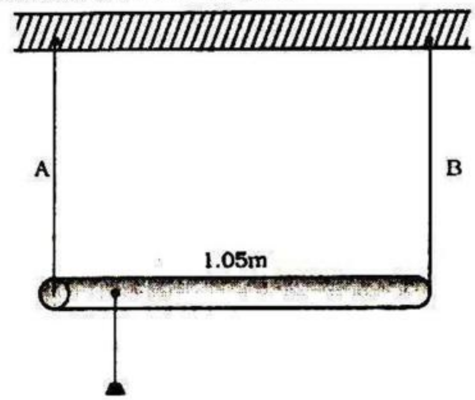
$$A = \frac{\pi d^2}{4}$$

Pressure at the tip of the anvil

$$P = \frac{F}{A}\text{ Pa} = \frac{50,000 \times 4}{\frac{22}{7} \times (5 \times 10^{-4})^2}\text{ Pa}$$

$$\boxed{P = 2.5 \times 10^{11} \text{ Pa}}$$

17. *A rod of length 1.05 m having negligible mass is supported at its ends by two wires of steel (wire A) and aluminium (wire B) of equal lengths as shown in figure. The cross-sectional areas of wires A and B are 1.0 mm² and 2.0 mm², respectively. At what point along the rod should a mass m be suspended in order to produce (a) equal stresses and (b) equal strains in both steel and aluminium wires.*



Sol. For steel wire A, $l_1 = l$; $A_1 = 1\text{ mm}^2$;

$$Y_1 = 2 \times 10^{11}\text{ Nm}^{-2}$$

For aluminium wire B, $l_2 = l$; $A_2 = 2\text{ mm}^2$

$$Y_2 = 7 \times 10^{10}\text{ Nm}^{-2}$$

(a) Let mass *m* be suspended from the rod at distance x from the end where wire A is connected. Let F_1 and F_2 be the tensions in two wires and there is equal stress in two wires, then

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \text{ or } \frac{F_1}{F_2} = \frac{A_1}{A_2} = \frac{1}{2}$$

Taking moment of forces about the point of suspension of mass from the rod, we have

$$F_1 x = F_2(1.05 - x)$$

or $\frac{1.05 - x}{x} = \frac{F_1}{F_2} = \frac{1}{2}$

or $2.10 - 2x = x$

or $x = 0.70\text{ m} = 70\text{ cm}$

(b) Let mass *m* be suspended from the rod at distance x from the end where wire A is connected. Let F_1 and F_2 be the tension in the wires and there is equal strain in the two wires *i.e.*

$$\frac{F_1}{A_1 Y_1} = \frac{F_2}{A_2 Y_2}$$

or $\frac{F_1}{F_2} = \frac{A_1}{A_2}\frac{Y_1}{Y_2} = \frac{1}{2} \times \frac{2 \times 10^{11}}{7 \times 10^{10}} = \frac{10}{7}$

As the rod is stationary, so $F_1 x = F_2 (1.05 - x)$

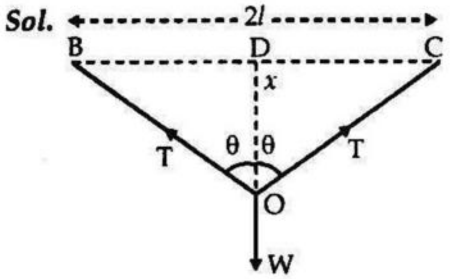
or $\frac{1.05 - x}{x} = \frac{F_1}{F_2} = \frac{10}{7}$

or $10x = 7.35 - 7x$

or $x = 0.4324 \text{ m} = 43.2 \text{ cm}$

18. *A mild steel wire of length 1.0 m and cross-sectional area 0.50×10^{-2} cm² is stretched, well within its elastic limit, horizontally between two pillars. A mass of 100 g is suspended from the mid-point of the wire. Calculate the depression at the mid-point.*

Sol.



Increase in length,

$\Delta l = BO + OC - 2l = 2BO - 2l$

$$= 2(l^2 + x^2)^{1/2} - 2l = 2l\left(1 + \frac{x^2}{l^2}\right)^{1/2} - 2l$$

$$= 2l\left[1 + \frac{x^2}{2l^2}\right] - 2l = \frac{x^2}{l}$$

Strain $= \frac{\Delta l}{2l} = \frac{x^2}{2l^2}$

If T is the tension in the string, then

$2\,T\cos\theta = W$ or $T = \frac{W}{2\cos\theta}$

Here, $\cos\theta = \frac{x}{OB} = \frac{x}{\sqrt{l^2 + x^2}}$

$$= \frac{x}{l\left(1 + \frac{x^2}{l^2}\right)^{1/2}} = \frac{x}{l\left(1 + \frac{x^2}{2l^2}\right)}$$

As $\frac{x^2}{2l^2} \ll 1$, so $1 + \frac{x^2}{2l^2} \approx 1$

So, $\cos\theta = \frac{x}{l}$ $\therefore$ $T = \frac{W}{2(x/l)} = \frac{Wl}{2x}$

stress $= \frac{T}{A} = \frac{Wl}{2Ax}$

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{Wl}{2Ax} \times \frac{2l^2}{x^2} = \frac{Wl^3}{Ax^3}$$

or, $x = l\left[\frac{W}{YA}\right]^{1/3}$

$$= 0.5\left[\frac{0.1 \times 10}{2 \times 10^{11} \times 0.5 \times 10^{-6}}\right]^{1/3}$$

$= 1.074 \times 10^{-2} \text{ m} = 1.074 \text{ cm}$

19. *Two strips of metal are riveted together at their ends by four rivets, each of diameter 6.0 mm. What is the maximum tension that can be exerted by the riveted strip if the shearing stress on the rivet is not to exceed 6.9×10^7 Pa? Assume that each rivet is to carry one quarter of the load.*

Sol. Here, $r = 6/2 = 3 \text{ mm} = 3 \times 10^{-3} \text{ m}$

Max. stress $= 6.9 \times 10^7$ Pa

Max. load on a rivet = Max. stress × area of cross section

$= 6.9 \times 10^7 \times (22/7) \times (3 \times 10^{-3})^2$

$\therefore$ Maximum tension

$$= 4\left[6.9 \times 10^7 \times \frac{22}{7} \times 9 \times 10^{-6}\right]$$

$= 7.8 \times 10^3$ N

20. *The Marina trench is located in the Pacific Ocean, and at one place it is nearly eleven km beneath the surface of water. The water pressure at the bottom of the trench is about 1.1×10^8 Pa. A steel ball of initial volume 0.32 m³ is dropped into the ocean and falls to the bottom of the trench. What is the change in the volume of the ball when it reaches to the bottom?*

Sol. Here, $p = 1.1 \times 10^8$ Pa;

$V = 0.32 \text{ m}^3$;

$K = 1.6 \times 10^{11}$ Pa

$$\Delta V = \frac{pV}{K} = \frac{(1.1 \times 10^8) \times 0.32}{1.6 \times 10^{11}}$$

$= 2.2 \times 10^{-4} \text{ m}^3$

Q1. The ratio of radii of two wires of same material is 2 :1. If these wires are stretched by equal force, what is the ratio of stresses produced in them ?

Ans. $r_1 : r_2 = 2 : 1$; $F_1 = F_2 = F$

Stress (S) $= \dfrac{\text{force}}{\text{area}} = \dfrac{F}{\pi r^2}$ or $S \propto \dfrac{1}{r^2}$

Hence $\dfrac{S_1}{S_2} = \dfrac{r_2^2}{r_1^2} = \left(\dfrac{1}{2}\right)^2 = \dfrac{1}{4}$

Q2. Find the force required to stretch a steel wire 1 sq. cm in cross section to double its length. Y for steel = 2×10^{11} Nm^{-2}.

Ans. $a = 1$ sq. cm $= 10^{-4}$ m^2; $\Delta l = 2l - l = l$

$$F = \frac{Y a \Delta l}{l} = \frac{2 \times 10^{11} \times 10^{-4} \times l}{l} = 2 \times 10^7 \text{ N}$$

Q3. A load of 2 kg produces an extension of 1 mm in a wire of 3 m long and 1 mm in diameter. Calculate the Young's modulus of elasticity of wire.

Ans.
$$Y = \frac{Mg}{\pi (D/2)^2} \times \frac{l}{\Delta l}$$
$$= \frac{2 \times 9.8 \times 3}{(22/7) \times (10^{-3}/2)^2 \times 10^{-3}}$$
$$= 7.48 \times 10^{10} \text{ N/m}^2$$

Q4. The length of a wire increases by 8 mm when a weight of 5 kg is hung at its lower end. If the conditions are the same, but the radius of the wire is doubled, what will be the increase in length?

Ans. $Y = \dfrac{F\,l}{a\,\Delta l}$ or $\Delta l = \dfrac{Fl}{aY} = \dfrac{Fl}{\pi r^2 Y}$

so, $\Delta l \propto \dfrac{1}{r^2}$ $\therefore$ $\dfrac{\Delta l_1}{\Delta l} = \dfrac{r^2}{r_1^2}$

or $\Delta l_1 = \Delta l \left(\dfrac{r^2}{r_1^2}\right) = 8 \times \dfrac{r}{(2r)^2} = 2\text{mm.}$

Q5. A structural steel rod has a radius of 10 mm and a length of 1 m. A 100 kN force stretches it along its length. Calculate

(a) the stress (b) elongation, and

(c) percentage strain on the rod.

Given that the Young's modulus of elasticity of the structural steel is 2.0×10^{11} Nm^{-2}.

Ans. r = 10 mm = 10×10^{-3} m = 10^{-2} m; l = 1 m

F = 100 kN = 10^5 N; Y = 2.0×10^{11} N/m^2

$$\text{Stress} = \frac{F}{A} = \frac{F}{\pi r^2} = \frac{10^5}{(22/7) \times (10^{-2})^2}$$
$$= 3.18 \times 10^8 \text{ Nm}^{-2}$$

$$\text{Elongation, } \Delta l = \frac{(F/A)\,l}{Y} = \frac{(3.18 \times 10^8) \times 1}{2 \times 10^{11}}$$
$$= 1.59 \times 10^{-3} \text{ m} = 1.59 \text{ mm}$$

$$\text{The strain} = \frac{\Delta l}{l} = \frac{1.59 \times 10^{-3}}{1} = 1.59 \times 10^{-3}$$

Percentage strain in rod

$= 1.59 \times 10^{-3} \times 100 = 0.159\%.$

Q6. Find the maximum length of a steel wire that can hang without breaking.

Breaking stress = 7.9×10^{12} dyne/cm^2

Density of steel = 7.9 g/cc.

Ans. Breaking stress $= \dfrac{\text{weight of wire}}{\text{area}}$

$= \dfrac{(Al)\,\rho\, g}{A} = l \rho g$

or $l = \dfrac{\text{breaking stress}}{\rho g} = \dfrac{7.9 \times 10^{12}}{7.9 \times 980}$

$= 1.02 \times 10^9$ cm.

Q7. The Young's modulus for a metal is given as 2.0×10^{11} Nm^{-2}. If the interatomic spacing for the metal is 2.8 Å, find the increase in the interatomic spacing for a force of 10^9 Nm^{-2} and the force constant.

Ans. $\dfrac{F}{A} = 10^9$ Nm^{-2}

As $Y = \dfrac{F}{A} \times \dfrac{l}{\Delta l}$

So, $\Delta l = \dfrac{F}{A} \times \dfrac{l}{Y} = 10^9 \times \dfrac{2.8 \times 10^{-10}}{2 \times 10^{11}}$

$= 1.4 \times 10^{-12}$ m

$= 0.014$ Å

As distance between two atoms is l, then area of one chain of atoms, $A = l \times l = l^2$

$$\therefore Y = \frac{F}{A} \times \frac{l}{\Delta l} = \frac{F}{\Delta l} \times \frac{l}{A} = \frac{F}{\Delta l} \times \frac{l}{l^2} = \frac{F}{\Delta l} \times \frac{1}{l}$$

$$\therefore \text{force constant, } k = \frac{F}{\Delta l} = Yl$$

$$= 2.0 \times 10^{11} \times (2.8 \times 10^{-10}) = 56 \text{ Nm}^{-1}$$

Q8. Four identical cylindrical columns of steel support a big structure of mass 50,000 kg. The inner and outer radii of each column are 30 cm and 40 cm respectively. Calculate the compressional strain of each column. Young's modulus of steel is 2.0×10^{11} Pa. Assume the load distribution to be uniform.

Ans. Area of cross section of each column

$$a = \pi(r_2^2 - r_1^2) = \pi[(0.4)^2 - (0.3)^2] = 0.07\pi \text{ m}^2$$

Weight of the structure

$$= Mg = 50000 \times 9.8 \text{ N}$$

The weight is equally shared by 4 columns.

$\therefore$ Compressional force on one column,

$$F = \frac{50000 \times 9.8}{4} \text{N}$$

$$Y = \frac{F/a}{\text{compressional strain}}$$

$$\therefore \text{Compressional strain} = \frac{F}{aY}$$

$$= \frac{50000 \times 9.8/4}{(\pi \times 0.07) \times 2.0 \times 10^{11}} = 2.785 \times 10^{-6}$$

Q9. A mass of 100 g is attached to the end of a rubber string 49 cm long and having an area of cross section 20 sq. mm. The string is whirled round horizontally at a constant speed of 40 r.p.s. in a circle of radius 51 cm. Find Young's modulus of rubber.

Ans. When the mass attached at the end of rubber string is whirled into a horizontal circle, the restoring force in the rubber string is equal to centripetal force.

$$= mr\omega^2 = 100 \times 51 \times (2 \times \pi \times 40)^2 \text{ dyne}$$

$$Y = \frac{F \times l}{a \times \Delta l} = \frac{(100 \times 51 \times 4\pi^2 \times 1600) \times 49}{(20 \times 10^{-2}) \times (51 - 49)}$$

$$= 3.95 \times 10^{10} \text{ dyne cm}^{-2}$$

$$= 3.95 \times 10^{9} \text{ Nm}^{-2}$$

Q10. Two parallel steel wires A and B are fixed to rigid support at the upper ends and subjected to the same load at the lower ends. The lengths of the wires are in the ratio 4 : 5 and their radii are in the ratio 4 : 3. The increase in the length of the wire A is 1 mm. Calculate the increase in the length of the wire B.

Ans. For wire A,

$F_1 = F,\ a_1 = \pi(4r)^2,\ l_1 = 4l,\ Y_1 = Y,\ \Delta l_1 = 1$ mm

For wire B,

$F_2 = F,\ a_2 = \pi(3r)^2,\ l_2 = 5l,\ Y_2 = Y,\ \Delta l_2 = ?$

$$Y = \frac{F_1}{a_1} \times \frac{l_1}{\Delta l_1} = \frac{F_2}{a_2} \times \frac{l_2}{\Delta l_2}$$

$$\therefore \Delta l_2 = \frac{F_2}{F_1} \times \frac{a_1}{a_2} \times \frac{l_2}{l_1} \times \Delta l_1$$

$$= 1 \times \left(\frac{4}{3}\right)^2 \times \frac{5}{4} \times 1 = 2.22 \text{ mm}$$

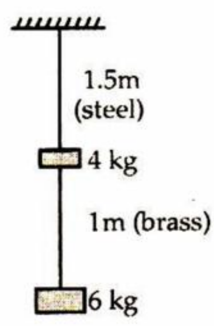
Q11. Two wires of equal cross-section but one made of steel and the other copper are joined end to end. When the combination is kept under tension, the elongation in the two wires are found to be equal. Find the ratio between the lengths of steel and copper wires. Given, Young's moduli of steel and copper are 2.0×10^{11} Nm^{-2} and 1.1×10^{11} Nm^{-2} respectively.

Ans. $Y_s = \frac{F}{A} \times \frac{l_s}{\Delta l_s}$; $Y_c = \frac{F}{A} \times \frac{l_c}{\Delta l_c}$

$$\text{So} \quad \frac{Y_s}{Y_c} = \frac{l_s}{l_c} \times \frac{\Delta l_c}{\Delta l_s} = \frac{l_s}{l_c} \qquad (\because \Delta l_c = \Delta l_s)$$

$$\therefore \quad \frac{l_s}{l_c} = \frac{Y_s}{Y_c} = \frac{2.0 \times 10^{11}}{1.1 \times 10^{11}} = \frac{20}{11}$$

Q12. Compute the elongation of the steel wire and brass wire as shown.



Area of X-section of each wire = 0.049 cm^2.
$Y_{steel} = 2 \times 10^{11}$ Pa & $Y_{Brass} = 0.90 \times 10^{11}$ Pa, $g = 9.8\ ms^{-2}$.

Ans. For steel wire, $F = 4 + 6 = 10$ kgf $= 10 \times 9.8$ N

$$\Delta l = \frac{F \times l}{Y \times a} = \frac{10 \times 9.8 \times 1.5}{2 \times 10^{11} \times (0.049 \times 10^{-4})} = 1.5 \times 10^{-4}\ m$$

For brass wire, $F = 6$ kgf $= 6 \times 9.8$ N

$$\Delta l = \frac{6 \times 9.8 \times 1}{0.90 \times 10^{11} \times (0.049 \times 10^{-4})} = 1.33 \times 10^{-4}\ m$$

Q13. A steel wire of 2 mm diameter is stretched between two clamps, when its temperature is 40°C. Calculate the tension in the wire when its temperature falls to 30°C.

Given, coefficient of linear expansion of steel = 11×10^{-6}/°C and Y for steel = 21×10^{11} dyne/cm^2.

Ans. $\Delta l = l\alpha(\theta_2 - \theta_1) = l \times (11 \times 10^{-6}) \times (40 - 30)$ cm.

$$F = \frac{Y\,a\,\Delta l}{l} = \frac{Y\pi r^2 \Delta l}{l}$$

Substituting the values, $F = 7.26 \times 10^6$ dyne

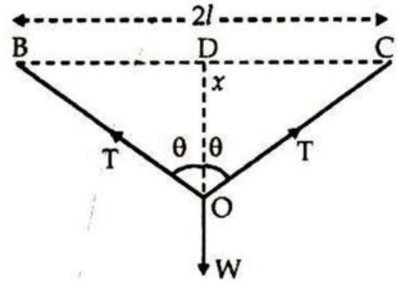
Q14. A wire of cross-sectional area A is stretched horizontally between two clamps located at a distance $2l$ metres from each other. A weight W kg is suspended from the mid point of the wire. If the vertical distance through which the mid point of the wire moves down be $x < l$, then find

(i) the strain produced in the wire

(ii) the stress in the area

(iii) If Y is the Young's modulus of wire, then find the value of x.

Ans.



Increase in length,

$$\Delta l = BO + OC - 2l = 2BO - 2l$$

$$= 2(l^2 + x^2)^{1/2} - 2l = 2l\left(1 + \frac{x^2}{l^2}\right)^{1/2} - 2l$$

$$= 2l\left[1 + \frac{x^2}{2l^2}\right] - 2l = \frac{x^2}{l}$$

$$\text{Strain} = \frac{\Delta l}{2l} = \frac{x^2}{2l^2}$$

(ii) If T is the tension in the string, then

$$2\,T\cos\theta = W \quad \text{or} \quad T = \frac{W}{2\cos\theta}$$

Here, $\cos\theta = \frac{x}{OB}$

$$= \frac{x}{\sqrt{l^2 + x^2}} = \frac{x}{l\left(1 + \frac{x^2}{l^2}\right)^{1/2}} = \frac{x}{l\left(1 + \frac{x^2}{2l^2}\right)}$$

As $\frac{x^2}{2l^2} \ll 1$, so $1 + \frac{x^2}{2l^2} \approx 1$

So, $\cos\theta = \frac{x}{l}$

$$\therefore \quad T = \frac{W}{2(x/l)} = \frac{W\,l}{2x}$$

Hence, $\text{stress} = \frac{T}{A} = \frac{W\,l}{2\,Ax}$

(iii) $Y = \frac{\text{stress}}{\text{strain}} = \frac{W\,l}{2A\,x} \times \frac{2l^2}{x^2} = \frac{Wl^3}{Ax^3}$

or $x = l\left[\frac{W}{YA}\right]^{1/3}$

Q15. A sphere contracts in volume by 0.02% when taken to the bottom of sea one km deep. Find bulk modulus of the material of sphere. Density of sea water is 1000 kg/m^3.

Ans. $\frac{\Delta V}{V} = \frac{0.02}{100}$,

p = 1000 m of water column $= 1000 \times 10^3 \times 9.8$ Pa

$$K = \frac{p}{\Delta V/V} = \frac{1000 \times 10^3 \times 9.8}{0.02/100} = 4.9 \times 10^{10}\ Pa$$

Q16. What will be the density of lead under a pressure of 20,000 N/cm^2? Density of lead is 11.4 g/cm^3 and the bulk modulus of lead is 0.80×10^{10} N/m^2.

Ans. $\Delta p = 20{,}000\ N/cm^2$

$= 20{,}000 \times 10^4\ N/m^2 = 2 \times 10^8\ N/m^2$

$$K = \frac{V\Delta p}{\Delta V}$$

$$\text{or } \Delta V = \frac{V\Delta p}{K} = V \times \frac{2\times10^8}{0.80\times10^{10}} = \frac{V}{40}$$

$$\therefore \text{New volume, } V' = V - \Delta V = \frac{39\ V}{40}$$

Let ρ' be the new density of lead, under the effect of pressure applied. As mass of the lead will remain the same, so $V\rho = V'\rho'$

$$\rho' = \frac{V\rho}{V'} = \frac{V\times 11.4}{39\ V/40} = 11.7\ g/cm^3.$$

Q17. The average depth of Indian ocean is about 3000 m. Calculate the percentage fractional compression of water at the bottom of the ocean, given that the bulk modulus of water is 2.2×10^9 Pa; $g = 10\ m/s^2$.

($1\ Pa = 1\ N/m^2$)

Ans. Hydrostatic pressure, $p = h\rho g$

$= 3000 \times 10^3 \times 10\ N/m^2$

$$\text{Fractional compression; } \frac{\Delta V}{V} = \frac{p}{k}$$

$$= \frac{3000\times10^3\times10}{2.2\times10^9} = 1.36\times10^{-2}$$

$$\%\ \text{Fractional compression} = \frac{\Delta V}{V}\times100$$

$= 1.36 \times 10^{-2} \times 100 = 1.36\%$

Q18. A cube is subjected to a pressure of $5 \times 10^5\ N/m^2$. Each side of the cube is shortened by 1%. Find volumetric strain and bulk modulus of elasticity of cube.

Ans. Let l be the initial length of each side of cube.

$$\text{Final length} = \left(1-\frac{1}{100}\right)l = \frac{99}{100}l$$

Initial volume, $V_i = l^3$

$$\text{Final volume, } V_f = \left(\frac{99}{100}l\right)^3$$

$\therefore$ Change in volume, $\Delta V = V_f - V_i$

$$= \left(\frac{99}{100}l\right)^3 - l^3$$

$$\text{Volumetric strain} = \frac{\Delta V}{V} = \left(\frac{99}{100}\right)^3 - 1$$

$$= \left(1-\frac{1}{100}\right)^3 - 1 = -\frac{3}{100} = -0.03$$

Normal stress = increase in pressure

$= 5 \times 10^5$

$$\therefore \text{Bulk modulus of elasticity, } K = \frac{5\times10^5}{0.03}$$

$= 1.67 \times 10^7\ N/m^2$

Q19. A 5 cm cube has its upper face displaced by 0.2 cm by a tangential force of 8 N. Calculate the shearing strain, shearing stress and modulus of rigidity of the material of cube.

Ans. $L = 5\ cm = 5 \times 10^{-2}\ m;$

$\Delta L = 0.2\ cm = 0.2 \times 10^{-2}\ m;$

$F = 8\ N;$

$$\text{Shearing strain} = \frac{\Delta L}{L} = \frac{0.2}{5} = 0.04$$

$$\text{Shearing stress} = \frac{F}{L\times L} = \frac{8}{(5\times10^{-2})^2}$$

$= 3200\ N/m^2$

Modulus of rigidity,

$$\eta = \frac{\text{shearing stress}}{\text{shearing strain}} = \frac{3200}{0.04}$$

$= 8 \times 10^4\ N/m^2$

Q20. A rubber cube of each side 7 cm has one side fixed, while a tangential force equal to the weight of 300 kgf is applied to the opposite face. Find the shearing strain produced and the distance through which the strained side moves. The modulus of rigidity for rubber is $2 \times 10^7\ dyne/cm^2$. Take $g = 10\ m/s^2$.

Ans. $L = 7\ cm = 7\times10^{-2}\ m; F = 300\ kgf = 300\times10\ N$

$\eta = 2 \times 10^7\ dyne/cm^2 = 2 \times 10^6\ N/m^2$

$$\text{As,} \quad \eta = \frac{F/a}{\theta}$$

$$\text{or} \quad \theta = \frac{F}{a\eta} = \frac{F}{L^2\eta}$$

$$= \frac{300\times10}{(7\times10^{-2})^2\times2\times10^6} \approx 0.3\ \text{rad}$$

$$\theta = \frac{\Delta L}{L}$$

or $\quad \Delta L = L\theta = 7 \times 0.3 = 2.1\ cm$

Q21. A metal cube of side 10 cm is subjected to a shearing stress of $10^4\ Nm^{-2}$. Calculate the modulus of rigidity if the top of the cube is displaced by 0.05 cm w.r.t. its bottom.

Ans. Here, $\Delta L = 0.05$ cm ; $L = 10$ cm

Shearing stress $= 10^4$ Nm^{-2};

Shearing strain $= \Delta L/L$

$= 0.05/10 = 0.005$

$$\text{Modulus of Rigidity}, \eta = \frac{\text{Shearing stress}}{\text{Shearing strain}}$$

$$= \frac{10^4}{0.005} = 2 \times 10^6 \text{ Nm}^{-2}$$

Q22. Two wires of same radius and length are subjected to the same load. One wire is of steel and the other is of copper. If the Young's modulus of steel is twice that of copper, find the ratio of elastic energy stored per unit volume in steel to that of copper.

Ans. Elastic energy per unit volume,

$$u = \frac{1}{2}\text{stress} \times \text{strain} = \frac{1}{2}\frac{(\text{stress})^2}{Y}$$

or $u \propto \frac{1}{Y}$; $\frac{u_s}{u_b} = \frac{Y_b}{Y_s} = \frac{1}{2}$

Q23. A steel wire of 4 m is stretched through 2 mm. The cross-sectional area of the wire is 2 mm^2. If Young's modulus of steel is 2×10^{11} Nm^{-2}, find

(i) the energy density of the wire.

(ii) the elastic potential energy stored in the wire.

Ans. Energy density, $u = \frac{1}{2}$ stress × strain

$$= \frac{1}{2} Y \times (\text{strain})^2 = \frac{1}{2} Y \times \left(\frac{\Delta l}{l}\right)^2$$

$$= \frac{1}{2} \times 2 \times 10^{11} \left[\frac{2 \times 10^{-3}}{4}\right]^2$$

$= 2.5 \times 10^4$ Jm^{-3}

Elastic potential energy, U

$=$ Energy density × volume $= u \times A \times l$

$= 2.5 \times 10^4 \times (2 \times 10^{-6}) \times 4.0 = 0.2$ J

Q24. A rubber cord catapult has a cross-sectional area 1 mm^2 and total unstretched length 10 cm. It is stretched to 12 cm and then released to project a stone of mass 500 g. Find the tension in the cord. Also find the velocity of projection of the stone. Take Young's modulus for rubber as 5×10^8 Nm^{-2}.

Ans. $\Delta l = 12 - 10 = 2$ cm $= 0.02$ m;

$l = 10$ cm $= 0.1$ m; $a = 1$ mm$^2 = 10^{-6}$ m^2

As $Y = \frac{F}{a} \times \frac{l}{\Delta l}$ so tension, $F = \frac{Y a \Delta l}{l}$

$$= \frac{5 \times 10^8 \times 10^{-6} \times 0.02}{0.1} = 100 \text{ N}$$

Kinetic energy of stone = elastic potential energy

$$\frac{1}{2} m v^2 = \frac{1}{2} \times \text{tension} \times \text{extension}$$

$$v = \sqrt{\frac{\text{tension} \times \text{extension}}{m}}$$

$$= \sqrt{\frac{100 \times 0.02}{500/1000}} = 2 \text{ ms}^{-1}$$

Q25. A 45 kg boy whose leg bones are 5 cm^2 in area and 50 cm long, falls through a height of 2 m without breaking his leg bones. If the bones can withstand a stress of 0.9×10^8 Nm^{-2}, calculate the Young's modulus for the material of the bone. Use $g = 10$ ms^{-2}.

Ans. Here, $m = 45$ kg; $h = 2$ m;

$L = 0.50$ m; $A = 5 \times 10^{-4}$ m^2

Loss in gravitational energy

= gain in elastic energy in both leg bones

So, $mgh = 2 \times \left[\frac{1}{2} \times \text{stress} \times \text{stain} \times \text{volume}\right]$

Volume $= AL = 5 \times 10^{-4} \times 0.50 = 2.5 \times 10^{-4}$ m^3

$\therefore 45 \times 10 \times 2$

$$= 2 \times \left[\frac{1}{2} \times 0.9 \times 10^8 \times \text{strain} \times 2.5 \times 10^{-4}\right]$$

or $$\text{strain} = \frac{45 \times 10 \times 2}{0.9 \times 2.5 \times 10^4} = 0.04$$

$$\therefore Y = \frac{\text{stress}}{\text{strain}} = \frac{0.9 \times 10^8}{0.04} = 2.25 \times 10^9 \text{ Nm}^{-2}$$

Q26. A lift is tied with thick iron wires and its mass is 1000 kg. The maximum acceleration of lift is 1.2 ms^{-2} and the maximum safe stress is 1.4×10^8 Nm^{-2}. Find the minimum diameter of the wire. $g = 9.8$ ms^{-2}.

Ans. The tension in the rope is,

$F = m(g + a) = 1000\,(9.8+1.2) = 11{,}000$ N

$$\text{Stress} = \frac{F}{A} = \frac{F}{\pi (D/2)^2}$$

or $$D^2 = \frac{4F}{\pi \times \text{stress}} = \frac{4 \times 11000 \times 7}{22 \times 1.4 \times 10^8} = \frac{1}{10^4}$$

$D = 0.01$ m