PRESSURE

Pressure is generally defined as "force per unit area", which is wrong. Pressure is defined as

pressure is the force per unit area acting *normally* on an infinitesimally (i.e. very) small element of area.

Pressure is a scalar physical quantity having no

$$\boxed{P_{avg} = \frac{F}{A}}$$

$$\boxed{P = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} = \frac{dF}{dA}}$$

where P = pressure
F = force
A = area

Units of pressure

SI unit is Pa or pascal.

1 Pa = 1 N/m². Other related units are

1 atm = 1.01×10^5 Pa = 760 torr
= 14.7 lb/in²

1 torr = pressure exerted by 1 mm of mercury column = 133 Pa.

1 bar = 10^5 Pa

1 millibar = 10^2 Pa

The dimensions of pressure are $[ML^{-1}T^{-2}]$.

Practical applications of the concept of pressure

Since pressure is the normal force per unit area, it follows that a given force will exert different pressures if it acts over different areas. This funda has many important applications in practice, such as:

(i) **Suitcases are provided with 'broad' handles,** so that small pressure is exerted on our hand while carrying them.

(ii) **Pins and nails are made with 'pointed' ends,** so that they have smallest area of contact with the given surface. Thus, when force is applied over the head of a pin or a nail, it exerts a 'large' pressure on the surface and penetrates into the surface. This is why it is easier to cut with a sharp knife than with a blunt one.

(iii) **Railways tracks are laid on 'large-sized' wooden sleepers,** so that the weight (thrust) of the train is spread over a large area. This reduces pressure on ground which is prevented from yielding.

(iv) **It is painful to walk barefoot on pebbles,** because our body weight is supported on very small areas of the pebbles which exert a large reactionary pressure on the portion of our feet in contact.

DENSITY

Density, ρ of a substance is defined as its mass per unit volume. If it is non-uniform, then it is defined as

$$\rho = \frac{\Delta m}{\Delta V}$$

where ΔV is a small volume of the substance around the point at which density is required. Δm is its mass.

Density is scalar. Its SI unit is kg/m³.

The dimensions of density are $[ML^{-3}]$.

Table 10.01 gives values of densities of some substances.

Density of Mixtures

(i) Case of two liquids, each of the same volume V but of different densities ρ_1 and ρ_2 mixed together.

$$\text{Total mass} = V\rho_1 + V\rho_2$$

$$\text{Total volume} = 2V$$

$$\text{Density of mixture,}\quad \rho = \frac{V\rho_1 + V\rho_2}{2V}$$

or $$\boxed{\rho = \frac{\rho_1 + \rho_2}{2}}$$

(ii) Case of two liquids, each of same mass m but of different densities ρ_1 and ρ_2 mixed together.

$$\text{Total mass} = 2m$$

$$\text{Total volume} = \frac{m}{\rho_1} + \frac{m}{\rho_2}$$

$$\text{Density of mixture,}\quad \rho = \frac{2m}{\frac{m}{\rho_1} + \frac{m}{\rho_2}}$$

or $$\boxed{\rho = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2}}$$

(iii) Case of two liquids of different masses m_1 and m_2 mixed together. Let ρ_1 and ρ_2 be their respective densities.

$$\text{Total mass} = m_1 + m_2$$

$$\text{Total volume} = \frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}$$

$$\text{Density of mixture, } \rho = \frac{m_1 + m_2}{\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}}$$

(iv) Case of two liquids of different volumes V_1 and V_2 mixed together. Let ρ_1 and ρ_2 be their respective densities.

$$\text{Total mass} = V_1\rho_1 + V_2\rho_2$$

$$\text{Total volume} = V_1 + V_2$$

$$\text{Density of mixture, } \rho = \frac{V_1\rho_1 + V_2\rho_2}{V_1 + V_2}$$

Density of gases varies considerably with pressure but not of liquids, because while gases are readily compressible, liquids are not.

RELATIVE DENSITY

Specific gravity (Sp. Gr.) is the modern name of ***relative density*** (RD). It is the density of a substance as compared to the density of water.

$$\text{Sp. Gr.} = \text{RD} = \frac{\text{Density of the substance}}{\text{Density of water}}$$

How is relative density measured

It should be noted that density or relative density is not of any object. It is of the material of the object. To determine the relative density of a given substance or material, take any object made out of it. Weigh this object in a normal way. It is called its *weight in air*. Then three cases arise as given below.

Case I–Density of the given material is more than the density of water.

Immerse the object completely in water and measure its weight.

R.D. of the material

$$= \frac{\text{Density of the material}}{\text{Density of water}}$$

$$= \frac{\dfrac{\text{Mass of object}}{\text{Volume of object}}}{\dfrac{\text{Mass of equal volume of water}}{\text{Corresponding volume of water i.e. vol. of object}}}$$

$$= \frac{\text{Mass of object}}{\text{Mass of equal volume of water}} \times \frac{g}{g}$$

$$= \frac{\text{Weight of object in air}}{\text{Weight of equal volume water}}$$

$$= \frac{\text{Weight of object in air}}{\text{Upthrust (by Archimedes principle)}}$$

$$\therefore \quad \text{R.D.} = \frac{\text{Weight of object in air}}{\text{Loss of weight of object in water}}$$

It can be also be written as below

$$\text{R.D.} = \frac{\text{Wt. in air}}{(\text{Wt. in air}) - (\text{Wt. in water})}$$

Case II–Density of the given material is equal to the density of water.

In this case, the object will stay in water in equilibrium at which ever height you place it gently in water. In such cases, R.D. = 1

Case III–Density of the given material is less than the density of water.

In such cases, you cannot immerse the object completely in water and let it remain there by itself. The object will come up and start floating. In such cases, find out the volume of object which remains immersed in water. Then,

$$\text{R.D.} = \frac{\text{Volume of object immersed in water}}{\text{Total volume of object}}$$

Because the R.D. of ice is 0.9 approximately, 90% of the total volume of an ice-berg floating in water would remain immersed inside the water. Only 10% will be visible.

PASCAL'S LAW

It can be stated in any of the following ways.

The pressure in a fluid at rest is same at all points if we ignore gravity.

or

Pressure applied to an enclosed fluid is transmitted undiminished to all parts of the fluid and to the walls of the container.

or

A liquid exerts equal pressure in all directions.

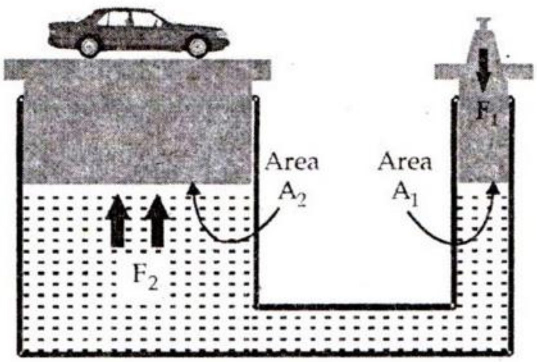
Applications of Pascal's Law

1. Hydraulic lifts, presses/Bramhas press etc.

Hydraulic lift is one of the most important applications of Pascal's law. It is used to lift heavy objects, such as vehicles at a service station.

A hydraulic lift consists of two cylinders connected to each other with a pipe. The two cylinders are of different areas of cross-sections and are provided with frictionless pistons as shown in Fig. Suppose that a force F_1 is applied on the smaller piston of cross-sectional area A_1. Then, pressure exerted over the liquid,

$$P = \frac{F_1}{A_1}$$



According to Pascal's law, the same pressure is transmitted to the larger piston of cross-sectional area A_2. Then, force transmitted to larger piston is given by

$$F_2 = P \times A_2$$

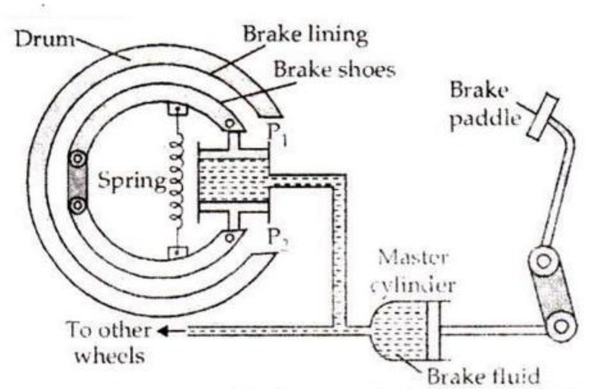
$$F_2 = \frac{F_1}{A_1} \times A_2$$

Since A_2 is greater than A_1, force F_2 on the larger piston will also be much larger than the force F_1 applied on the smaller piston. A heavy load placed on the larger piston is then easily lifted.

2. Hydraulic Brakes

When the driver of the vehicle puts pressure on the brake paddle, the lever system moves a piston into the master cylinder containing brake fluid.

The brake fluid from the master cylinder is led through strong pipes to cylinders provided with pistons (P_1, P_2) of larger cross-sectional area. Thus,



a small force applied over the brake paddle is transmitted by the brake fluid as a large force to the piston for brake and as a result, the brake shoes open tend to up. They press against the brake linings of the drums of the wheels and bring them to rest.

When the paddle is released, 2 spring system brings the brake shoes back to their normal positions and the brake fluid returns to the master cylinder.

Example

In a hydraulic press, the two pistons are of diameters of 30.0 cm and 2.5 cm. Estimate the force exerted by the larger piston when 50.0 kg wt. is placed on smaller piston. When the stroke of the smaller piston is 4.0 cm, what is the distance through which the larger piston would move after 10 strokes.

Solution

$$A_1 = \pi\,(2.5/2)\ \text{cm}^2;$$
$$A_2 = \pi\,(30/2)^2\ \text{cm}^2;\ F_1 = 50\ \text{kg wt.}$$

$$\text{Now, } F_2 = \frac{F_1}{A_1} \times A_2 = \frac{50 \times \pi \times (30/2)^2}{\pi\,(2.5/2)^2}$$

$$= \frac{50 \times 30 \times 30}{2.5 \times 2.5} = 7200\ \text{kg wt.}$$

In one stroke

Input workdone = output workdone

$$\text{so} \quad F_1 l_1 = F_2 l_2$$

$$\text{or} \quad l_2 = \frac{F_1 l_1}{F_2} = \frac{50 \times 4}{7200} = 0.028$$

$\therefore$ Distance covered after 10 strokes

$$= 0.028 \times 10 = \mathbf{0.28\ cm.}$$

Examples of Liquid pressure

1. Tyres of heavy load-trucks are made wide

Obviously weight of entire truck falls on tyres. Further, pressure = $\frac{\text{Force}}{\text{Area}}$; hence if tyres are made of small area; the pressure on them will be very large. They will get damaged soon. However, if tyres have large area in contact with ground, there will be less pressure on them, hence they will remain intact, in good condition for a long time.

2. Foundations of high rising buildings are made wide

The weight of entire buildings falls on the foundation. If the foundation is made wide, the area will be large, hence the pressure exerted by them on earth will be small. So the building will remain safe.

3. Divers wear metallic covers while going down into the sea

The water pressure increases with increase of depth of water. The increase in pressure is 1 atmosphere per 10 m increase of depth of water. If this order of pressure falls on the diver directly, he cannot tolerate it. Therefore, if a diver wants to go to a great depth, he wears the metallic cover. The pressure of water column falls on the metallic cover while the pressure inside the cover remains nearly equal to the atmospheric pressure.

4. Walls of a dam are made thick at the bottom and thin upwards

The pressure at a point inside a liquid depends on the depth of point from the free surface of liquid, therefore, the pressure is very high at the bottom of the dam. To take care of this pressure, the walls of dam are made thick at the bottom.

Proof of Pascal's Law

Let us imagine the volume of liquid which is inside a very small right angled prism.

Let it has uniform thickness t.

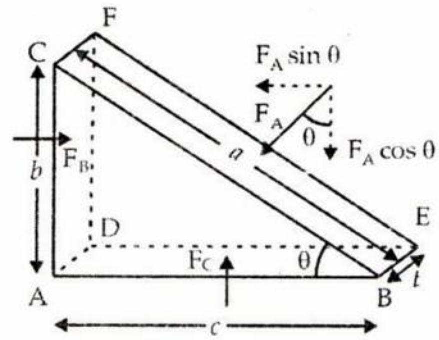
$\therefore$ Areas of the three faces of prism will be as

Face ADEB $= AB \times t = ct$

Face ADFC $= AC \times t = bt$

Face BCFE $= BC \times t = at$

Let F_C, F_B and F_A are the forces being acted on these three faces respectively. F_A can be resolved into two components $F_A \cos\theta$ and $F_A \sin\theta$.



Applying Newton's first law that as this prism is in equilibrium at rest, hence no net external force would be acting on it.

$$\therefore \quad F_C = F_A \cos\theta \qquad \text{... (i)}$$

$$F_B = F_A \sin\theta \qquad \text{... (ii)}$$

and P_A = Pressure on face BCFE

$$= \frac{F_A}{at} \qquad \text{... (iii)}$$

P_B = Pressure on face ADFC

$$= \frac{F_B}{bt} = \frac{F_A \sin\theta}{bt}$$

$$= \frac{F_A \times b/a}{bt} = \frac{F_A}{at} \qquad \text{... (iv)}$$

P_C = Pressure on face ADEB

$$= \frac{F_C}{ct} = \frac{F_A \cos\theta}{ct}$$

$$= \frac{F_A \times c/a}{ct} = \frac{F_A}{at} \qquad \text{... (v)}$$

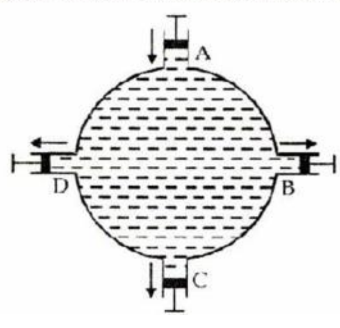
Thus from equations (iii), (iv) and (v) we get

$$P_A = P_B = P_C$$

This is what Pascal's law states.

Verification of Pascal's Law

To verify Pascal's law, take a spherical hollow vessel fitted with four small water tight pistons having same cross-sectional area as shown. The vessel is completely filled with water. Force is applied on one piston to push it inward. By this the pressure is exerted on water inside the vessel. This pressure is transmitted to all the other pistons,



The pistons B, C, D move outwards travelling the same distances. To stop the motion of these pistons, we have to exert equal force on each piston; the magnitude of each force being equal to the force applied on piston A. It means that the pressure is transmitted by the liquid equally, undiminished in all directions.

Variation of pressure with depth, effect of gravity

consider a small volume element in between the liquid at a depth y.

Let A = area of cross-section of this volume element.

dy = its thickness

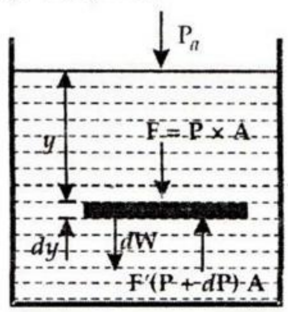
P_a – atmospheric (*i.e.* outside) pressure acting on the free surface of liquid.

F = Total force acting downward on the top face of the volume element.

$= P \times A$

F' = Total upward force acting on its bottom face.

$= (P + dP) \times A$



dW = weight of this volume element, acting downward.

$\therefore$ As this volume element is in equilibrium,

$$(P + dP) \times A - P \times A = dW$$

$$dP \times A = (A \times dy)\rho g$$

or $$dP = \rho g\, dy$$

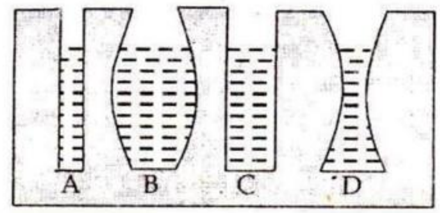
$$\therefore \int_{P_a}^{P} dP = \int_0^h \rho g\, dy$$

$$[P]_{P_a}^{P} = \rho g [y]_0^h$$

$$\Rightarrow \boxed{P = P_a + \rho g h}$$

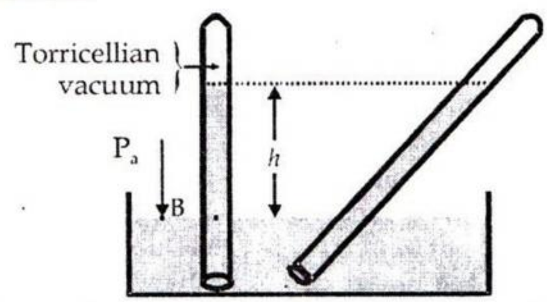
HYDROSTATIC PARADOX

The pressure at the bottom of a container will not depend upon the shape or size of the container. It will depend only upon the density of liquid (ρ), height of liquid column (h) and acceleration due to gravity (g). Thus pressures at the bottom of containers A, B, C and D in fig. will be exactly the same. This is called hydrostatic paradox.



Measurements of atmospheric pressure (Barometer)

The instrument or device which is used to measure atmospheric pressure is called a *barometer*. The first such device was invented by Torricelli



A long glass tube (of about 1 m length) closed at one end was filled completely with mercury and then was inverted in a trough containing mercury.

The mercury level in the tube falls and stands upto a certain height. The space above the mercury level does not contain air. This space is called the Torricellian vacuum. Even after all precautions. Torricelli observed that height h of mercury column in the tube remained he same (760 mm). Length of the tube did not matter.

Applying Pascal's law, pressure will be same at the same height.

$\therefore$ Pressure at A = Pressure at B

$$\boxed{P_a = \rho g h}$$

$\because$ Density of mercury $\rho = 13.6 \times 10^3\ kg/m^3$

$\therefore$ 1 mm height of Hg column will exert a pressure $= (13.6 \times 10^3) \times (9.81) \times 10^{-3}\ N/m^2$.

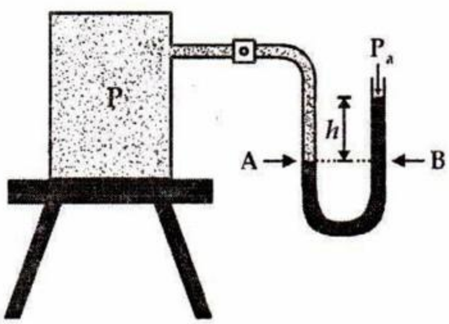
$= 133\ N/m^2$ or Pa.

This much pressure is called 1 torr.

U-tube manometer

(also called open tube manometer)

It is a simple U-tube with ends open. Some mercury is filled in it. In fig. P is a vessel containing some gas whose pressure is to be determined. It is connected as shown. The level of mercury in the two vertical limbs of the U-tube becomes as shown. The difference of height 'h' of mercury columns in the two limbs gives the pressure as shown below.



As levels A and B are at the same height, so

Pressure at A = Pressure at B

Required pressure, $P = P_a + \rho g h$

P is called ***absolute pressure***. $\rho g h$ is called ***gauge pressure***, P_g which is $P - P_a$.

Buoyant force. Whenever an object is placed in a fluid, the fluid exerts an upward force on it. This is called *buoyant force.*

Factors affecting the buoyant force :

(i) The buoyant force increases with increase of volume submerged

(ii) The buoyant force increases with increase of density of liquid

(iii) The buoyant force increases with increase of value of acceleration due to gravity

Archimedes principle states that if a solid is immersed in a fluid, partially or fully, it experiences a loss in its weight. This loss in its weight is equal to the weight of the fluid displaced by that solid.

Proof of Archimedes Principle. Let a solid cube is immersed completely in a fluid as shown.

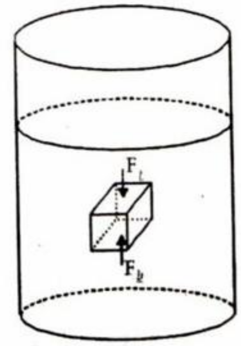
Let t, b = subscripts used for top & bottom faces

F, P = force, pressure.

f = subscript for fluid.

A = area of cross-section.

V = volume of cube.



h = height of cube.

ρ = density.

$\therefore$ Buoyant force on this cube

= Net upward force on this cube due to fluid pressure.

= (upward fluid force F_b on its bottom) – (downward fluid force F_t on its top face)

$= P_b \times A - P_t \times A = (P_b - P_t) \times A$

$= \rho_f g h A = \rho_f g V$

= weight of the fluid displaced.

This is what Archimedes principle states.

It will be applicable to any other body by simple extension of the idea.

Principle of Floatation. When a body is placed in a liquid, two vertical forces act on it.

(*a*) Weight of the body (W) acting downwards.

(*b*) Upthrust (W′) equal to the weight of the liquid displaced and acting upwards.

Case I : W < W′: The body sinks because downward force is greater than the upward force.

Case II: W = W′ : The body floats completely immersed in liquid. The apparent weight of the body in this case is zero.

Case III: W > W′: In this case, the body floats with a part of it out side the liquid.

1. *A 50 kg girl wearing high heel shoes balances on a single heel. The heel is circular with a diameter 1.0 cm. What is the pressure exerted by the heel on the horizontal floor ?*

Sol. We know that

$$\text{Pressure, } P = \frac{mg}{\pi (D/2)^2} = \frac{4mg}{\pi D^2}$$

$$P = \frac{4 \times 50 \times 9.8}{\left(\frac{22}{7}\right) \times (10^{-2})^2}$$

$$P = 6.2 \times 10^6 \text{ Pa. } \quad \textbf{Ans.}$$

2. *Toricelli's barometer used mercury. Pascal duplicated it using French wine of density 984 kg m^{-3}. Determine the height of the wine column for normal atmospheric pressure.*

Sol. $P = 0.76 \times (13.6 \times 10^3) \times 9.8$ ...(*i*)

Also, $P = h \times 984 \times 9.8$ ...(*ii*)

Equating the two equations

$h \times 984 \times 9.8 = 0.76\ (13.6 \times 10^3) \times g$

$$\Rightarrow \quad h = \frac{0.76 \times (13.6 \times 10^3)}{984}$$

$$h = 10.5 \text{ m} \quad \textbf{Ans.}$$

3 . *A vertical off-shore structure is built to wind stand a maximum stress of 10^9 Pa. Is the structure suitable for putting up on top of an oil well in the ocean ? Take the depth of the ocean to be roughly 3 km, and ignore ocean currents.*

Sol. Given $P = 10^9$ Pa, $h = 3$ km $= 3 \times 10^3$ m

$\rho = 1000$ kg m^{-3}

We know that

Pressure, $P = h\rho g$

$\Rightarrow \quad P = 3000 \times 1000 \times 9.8$

$P = 2.94 \times 10^7$ Pa

Since the pressure exerted by sea water is less than the maximum stress the structure can withstand, the structure is suitable for putting on the top of oil well.

4 . *A hydraulic automobile lift is designed to lift cars with a maximum mass of 3000 kg. The area of cross-section of the piston carrying the load is 425 cm^2. What maximum pressure would the smaller piston have to bear ?*

Maximum pressure on smaller piston

= Maximum pressure on bigger piston

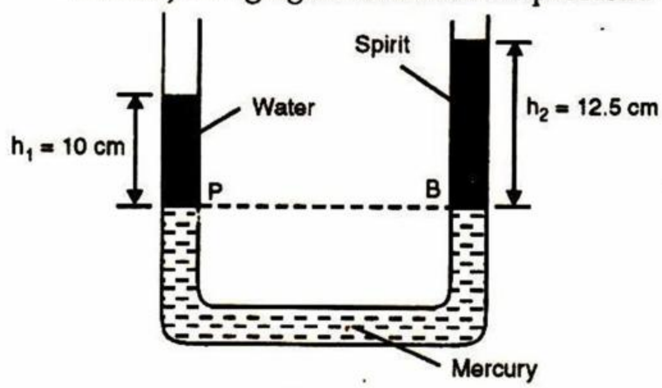
$$= \frac{\text{Max. load on bigger piston}}{\text{Area of cross-section of bigger piston}}$$

$$= \frac{3000 \times 9.8}{425 \times 10^{-4}} \text{ Nm}^{-2}$$

$$\boxed{= 6.917 \times 10^5 \text{ Pa}}$$

5 . *A U-tube contains water and methylated spirit separated by mercury. The mercury columns in the two arms are in level with 10.0 cm of water in one arm and 12.5 cm of spirit in the other. What is the specific gravity of spirit ?*

Sol. Adjoining figure describes the problem.



The pressure at the interfaces is marked *P* and *B*, are at the same height above the reference level.

This implies pressure exerted by water column at *P* must be equal to the pressure exerted by column of spirit at *B* *i.e.*,

$$h_1 \rho_1 g = h_2 \rho_2 g$$

$$\Rightarrow \quad \rho_2 = \frac{h_1}{h_2} \times \rho_1$$

$$\rho_2 = \frac{10}{12.5} \times 1$$

$$\boxed{\rho_2 = 0.8\ g\ \text{cm}^{-3}}$$

6 . *In the previous problem, if 15.0 cm of water and spirit each are further poured into the respective arms in the tube, what is the difference in the levels of mercury in the two arms ? (Specific gravity of mercury = 13.6)*

Sol. On pouring 15 cm of water and spirit in the respective arms,

$$h_1 = 10 + 15 = 25 \text{ cm}$$

and $$h_2 = 12.5 + 15 = 27.5 \text{ cm}$$

Pressure at the mercury -water interface is :

$$P_1 = h_1 \rho_1 g$$

$$\Rightarrow \quad P_1 = 25 \times 1 \times g = 25 \text{ g Pa}$$

Pressure at the mercury-spirit interface is

$$P_2 = h_2 \rho_2 g = 27.5 \times 0.8 \times g$$

$$P_2 = 22 \text{ g Pa}$$

Since pressure at water-mercury interface is more than the pressure at spirit mercury interface, the mercury will rise in the spirit arm. If 'h' is the difference in the levels of mercury in the two arms, then,

$$P_1 - P_2 = h \rho g$$

$$\Rightarrow \quad 25 \text{ g} - 22 \text{ g} = h \times 13.6 \times g$$

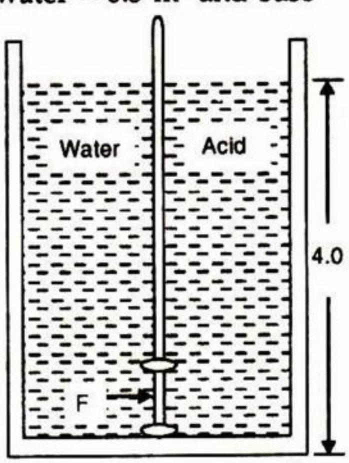
$$\Rightarrow \quad h = \frac{3}{13.6} \text{ cm}$$

$$\boxed{h = 0.22 \text{ cm}}$$

7 . *A tank with a square base of area 1.0 m² is divided by a vertical partition in the middle. The bottom of the partition has a small-hinged door of area 20 cm². The tank is filled with water in one compartment, and an acid (of relative density 1.7) in the other, both to a height of 4.0 m. Compute the force necessary to keep the door closed.*

Sol. Total base area = 1.0 m^2

$\therefore$ Base area for water = 0.5 m^2 and base area for acid = 0.5 m^2



Area of the hinged door = $20 \text{ cm}^2 = 20 \times 10^{-4} \text{ m}^2$

$\therefore$ Water pressure on the bottom = $h\rho g$

$$= 4 \times 10^3 \times 9.8 \text{ N/m}^2$$

and Acid pressure on the bottom

$$= 4 \times 1.7 \times 10^3 \times 9.8 \text{ N/m}^2$$

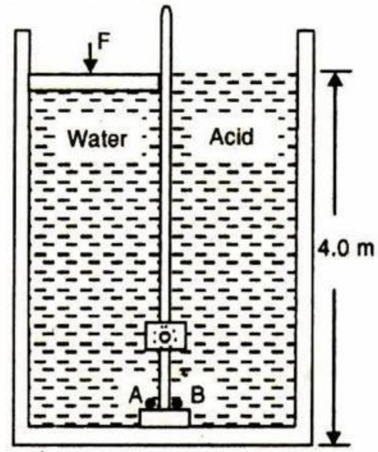
$\therefore$ Pressure difference on the two sides of the hinged door = $4 \times 10^3 \times 9.8\,(1.7 - 1) \text{ N/m}^2$.

$\therefore$ Force F required = $[4 \times 10^3 \times 9.8\,(1.7 - 1)] \times$ [Area of hinged door]

$$= [4 \times 10^3 \times 9.8 \times 0.7] \times [20 \times 10^{-4}] \text{ N}$$

$$= 54.88 \text{ N} \approx 55 \text{ N}.$$

SPECIAL NOTE : By just slight modification, this question can be made of IIT standard. Suppose we have to find out the value of the force F if it is to be applied on a massless piston placed on the water column as shown.



Let A and B are two points on the hinged door such that their height from the bottom of the vessel is neglible.

Pressure at A will due to two causes, namely

(*i*) due to water column of height 4.0 m

(*ii*) due to force F (whose value is to be determined) required to keep the hinged door just closed.

$\therefore$ Pressure at A

$$= (h\rho g) \text{ water} + \frac{\text{F}}{\text{Half of base area}}$$

$$= \left[4 \times 10^3 \times 9.8 + \frac{F}{0.5}\right] \text{N/m}^2$$

$\therefore$ Force at A

$$= \left[4 \times 10^3 \times 9.8 + \frac{F}{0.5}\right] \times \text{area of door.}$$

$$= \left[4 \times 10^3 \times 9.8 + \frac{F}{0.5}\right] \times 20 \times 10^{-4} \text{ N} \qquad \ldots(i)$$

Similarly force B = [Pressure due to acid column] × door area

$= [h\rho g]$ acid × door area

$= [4 \times 1.7 \times 10^3 \times 9.8] \times 20 \times 10^{-4}$ N ...(ii)

∴ For door to remain closed,

$$\left[4 \times 10^3 \times 9.8 + \frac{F}{0.5}\right] \times 20 \times 10^{-4}$$

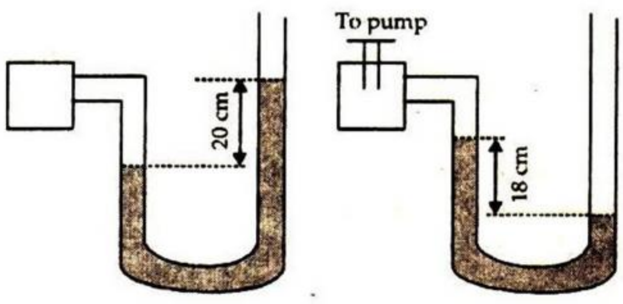
$$= [4 \times 1.7 \times 10^3 \times 9.8] \times 20 \times 10^{-4}$$

On solving, $F = 1.372 \times 10^4$ N.

8 . *A manometer reads the pressure of a gas in an enclosure as shown in Fig. (a). When a pump removes some of the gas, the manometer reads as in Fig. (b). The liquid used in the manometers is mercury and the atmospheric pressure is 76 cm of mercury.*

(a) Give the absolute and gauge pressure of the gas in the enclosure for cases (a) and (b), in units of cm of mercury.

(b) How would the levels change in case (b) if 13.6 cm of water (immiscible with mercury) is poured into the right limb of the manometer ? (Ignore the small change in the volume of the gas).



(a) (b)

Sol. (*a*) Here, atmospheric pressure, $P = 76$ cm of mercury

In Fig. (*a*), pressure head, $h = +20$ cm.

∴ Absolute pressure,

$P' = P + h = (76 + 20)$ cm of mercury.

Gauge pressure = $\boxed{h = +20 \text{ cm}}$

In Fig. (*b*), pressure head, $h' = -18$ cm

Absolute pressure $= P + h$

$P'' = 76 - 18$

$\boxed{P'' = 58 \text{ cm of mercury}}$

Gauge pressure $= h' = -18$ cm of mercury

⇒ $\boxed{h' = -18 \text{ cm}}$

(*b*) 13.6 cm of water added in right limb is equivalent to $\frac{13.6}{13.6} = 1$ cm of mercury column

⇒ $h_i = 1$ cm of mercury column.

Now, pressure at A

$P_A = P + h_i = 76 + 1$

$P_A = 77$ cm.

Let the difference in mercury levels in the two limbs be h_0 then pressure at B,

$P_B = 58 + h_0$

As $P_A = P_B$

∴ $77 = 58 + h_0$

⇒ $h_0 = 77 - 58$

$\boxed{h_0 = 19 \text{ cm of mercury}}$

9 . *During blood transfusion the needle is inserted in a vein where the gauge pressure is 2000 Pa. At what height must the blood container be placed so that blood may just enter the vein ?*

Density of blood = 1.06×10^3 kg/m^3.

Sol. Given $P = 2000$ Pa

$\rho = 1.06 \times 10^3$ kg m^3

We know that

$$h = \frac{P}{\rho g} = \frac{2000}{1.06 \times 10^3 \times 9.8}$$

$= 0.1925$ m. **Ans.**

10 . *A spring balance reads 10 kg when a bucket of water is suspended from it. What is the reading on the spring balance when*

(a) an ice cube of mass 1.5 kg is put into the bucket ?

(b) an iron piece of mass 7.8 kg suspended by another string is immersed with half its volume inside the water in the bucket ? (Relative density of iron = 7.8).

Sol. (*a*) When the ice is put into the bucket of water, then the total mass of the system suspended from the spring balance will be 10 kg + 1.5 kg = 11.5 kg, so the balance will read = 11.5 kg.

(*b*) Density of iron = relative density × density of water $= 7.8 \times 10^3$ kg/m^3

Volume of iron piece,

$$V = \frac{\text{mass}}{\text{density}} = \frac{7.8}{7.8 \times 10^3} = 10^{-3} \text{ m}^3$$

Volume of iron piece under water

$$\Rightarrow \quad \frac{V}{2} = \frac{1}{2} \times 10^{-3}\ \text{m}^3$$

Volume of water displaced

$$= \frac{V}{2} = 10^{-3}\ \text{m}^3 \times \frac{1}{2}$$

Weight of water displaced

$$= \left(\frac{1}{2} \times 10^{-3}\right) \times 10^3 \times 9.8\ \text{N} = 0.5\ \text{kg wt.}$$

This is the upward buoyant force experienced by iron piece due to water. The iron piece will exert an equal and opposite downward force on water. So, the reading of the balance will increase by 0.5 kg. Thus the spring balance will read

$$m = 10\ \text{kg} + 0.5\ \text{kg}$$

$$m = 10.5\ \text{kg} \quad \textbf{Ans.}$$