RAY OPTICS

1 Mark Questions

1. How does focal length of a lens change when red light incident on it is replaced by violet light? Give reason for your answer.

2. Name the physical quantity which remains same for microwaves of wavelength 1 mm and UV radiation of 1600 Å in vacuum.

3. Under what condition does a biconvex lens of glass having a certain refractive index act as a plane glass sheet when immersed in a liquid?

4. For the same value of angle of incidence, the angles of refraction in three media *A*, *B* and C are 15°, 25° and 35° respectively. In which medium would the velocity of light be minimum?

5. When monochromatic light travels from one medium to another, its wavelength changes but frequency remains the same. Explain.

6. The refractive index of diamond is much greater than that of glass. How does a diamond cutter make use of this fact?

7. If a ray of light propagates from a rarer to a denser medium, how does its frequency change?

8. State the criteria for the phenomenon of total internal reflection of light to take place.

9. A lens behaves as a converging lens in air and a diverging lens in water ($\mu = 4/3$). What will be the condition on the value of refractive index (μ) of the material of the lens?

10. A converging lens is kept coaxially in contact with a diverging lens; both the lenses being of equal focal lengths. What is the focal length of the combination?

11. When light travels from a rarer to a denser medium, the speed decreases. Does this decrease in speed imply a decrease in the energy carried by the light wave? Justify your answer.

12. A glass lens of refractive index 1.45 disappears when immersed in a liquid. What is the value of refractive index of the liquid?

13. State the conditions for the phenomenon of total internal reflection to occur.

14. Calculate the speed of light in a medium whose critical angle is 30°.

15. Why does the sky appear blue?

16. Under what condition does the formation of rainbow occur?

17. Two thin lenses of power + 6D and − 2D are in contact. What is the focal length of the combination?

18. Two thin lenses of power + 4 D and − 2D are in contact. What is the focal length of the combination?

19. Two thin lenses of power + 5D and − 2.5D are in contact. What is the focal length of the combination?

20. Why are convex mirrors used as side view mirrors in cars?

21. A converging lens of refractive index 1.5 is kept in a liquid medium having same refractive index. What would be the focal length of the lens in this medium?

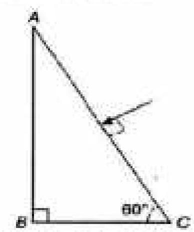
22. How does the power of convex lens vary, if the incident red light is replaced by violet light?

23. Explain giving reason why the sun looks reddish at sunrise or sunset?

24. A convex lens of refractive index 1.5 has a focal length of 18 cm in air. Calculate the change in its focal length when it is immersed in water of refractive index $\frac{4}{3}$.

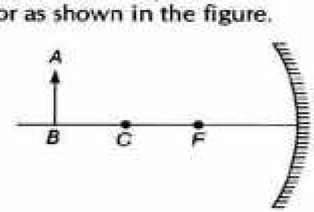
2 Marks Questions

25. Trace the path of a ray of light passing through a glass prism (*ABC*) as shown in the figure. If the refractive index of glass is $\sqrt{3}$, find out the value of the angle of emergence from the prism.



26. A ray of light, incident on an equilateral glass prism ($\mu_g = \sqrt{3}$) moves parallel to the base line of the prism inside it. Find the angle of incidence for this ray.

27. An object *AB* is kept in front of a concave mirror as shown in the figure.



(a) Complete the ray diagram showing the image formation of the object.

(b) How will the position and intensity of the image be affected if the lower half of the mirror's reflecting surface is painted black?

28. A beam of light converges at a point *P*. A concave lens of focal length 16 cm is placed in the path of this beam 12 cm from *P*. Draw a ray diagram and find the location of the point at which the beam would now converge.

29. Draw a diagram showing the formation of primary rainbow and explain at what angles the primary rainbow is visible.

30. The radii of curvature of the faces of a double convex lens are 10 cm and 15 cm. If focal length of the lens is 12 cm, find the refractive index of the material of the lens.

31. (a) The bluish colour predominates in clear sky.

(b) Violet colour is seen at the bottom of the spectrum when white light is dispersed by a prism.

State reasons to explain these observations.

32. A biconvex lens has a focal length 2/3 times the radius of curvature of either surface. Calculate the refractive index of lens material.

33. (a) Why does the sun appear reddish at sunset or sunrise?

(b) For which colour the refractive index of prism material is maximum and minimum?

34. Find the radius of curvature of the convex surface of a plano-convex lens, whose focal length is 0.3 m and the refractive index of the material of the lens is 1.5.

35. (a) Out of blue and red light which is deviated more by a prism? Give reason.

(b) Give the formula that can be used to determine refractive index of material of a prism in minimum deviation condition.

36. The following table gives the values of the angle of deviation, for different values of the angle of incidence, for a triangular prism :

Angle of incidence	33°	38°	42°	52°	60°	71°
Angle of deviation	60°	50°	46°	40°	43°	50°

(a) For what value of the angle of incidence, is the angle of emergence likely to be equal to the angle of incidence itself?

(b) Draw a ray diagram, showing the passage of a ray of light through this prism when the angle of incidence has the above value.

37. (a) State the principle on which the working of an optical fibre is based.

(b) What are the necessary conditions for this phenomenon to occur?

38. (a) What is the relation between critical angle and refractive index of a material?

(b) Does critical angle depend on the colour of light? Explain.

39. A ray of light passing through an equilateral triangular glass from air undergoes minimum deviation when angle of incidence is 3/4th of the angle of prism. Calculate the speed of light in the prism.

40. Calculate the distance of an object of height *h* from a concave mirror of focal length 10 cm, so as to obtain a real image of magnification 2.

41. A screen is placed 100 cm from an object. The image of the object on the screen is formed by a convex lens at two different locations, separated by 20 cm. Calculate the focal length of the lens used.

42. A double convex lens of glass of refractive index 1.6 has its both surfaces of equal radii of curvature of 30 cm each. An object of height 5 cm is placed at a distance of

12.5 cm from the lens. Calculate the size of the image formed.

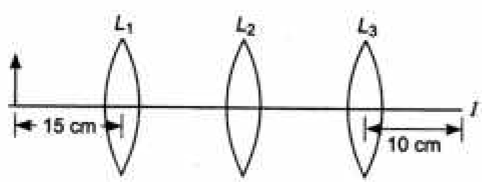
43. A convex lens of refractive index 1.5 has a focal length of 20 cm in air. Calculate the change in its focal length when it is immersed in water of refractive index 4/3.

44. A concave lens has the same radii of curvature for both sides and has a refractive index 1.6 in air. In the second case, it is immersed in a liquid of refractive index 1.4. Calculate the ratio of the focal lengths of the lens in the two cases.

3 Marks Questions

45. Define power of a lens. Write its units. Deduce the relation $\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$ for two thin lenses kept in contact coaxially.

46. You are given three lenses L_1, L_2 and L_3 each of focal length 10 cm. An object is kept at 15 cm in front of L_1, as shown. The final real image is formed at the focus I of L_3. Find the separation between L_1, L_2 and L_3.



47. A convex lens made up of glass of refractive index 1.5 is dipped, in turn, in

(i) a medium of refractive index 1.65

(ii) a medium of refractive index 1.33

(a) Will it behave as a converging or a diverging lens in the two cases?

(b) How will its focal length change in the two media?

48. Use the mirror equation to show that

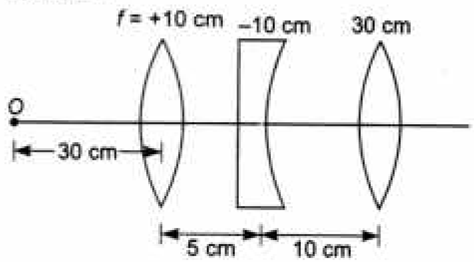
(a) An object placed between f and $2f$ of a concave mirror produces a real image beyond $2f$.

(b) A convex mirror always produces a virtual image independent of the location of the object.

(c) An object placed between the pole and focus of a concave mirror produces a virtual and enlarged image.

49. A converging lens has a focal length of 20 cm in air. It is made of a material of refractive index 1.6. It is immersed in a liquid of refractive index 1.3. Calculate its new focal length.

50. Find the position of the image formed of the object O by the lens combination given in the figure.



51. State the necessary conditions for producing total internal reflection of light. Draw ray diagrams to show how specially designed prisms make use of total internal reflection to obtain inverted image of the object by deviation rays?

(a) Through 90° and

(b) Through 180°.

52. With the help of suitable ray diagram, derive a relation between the object distance (u), image distance (v) and radius of curvature R for the convex spherical surface, when a ray of light travels from rarer to denser medium.

53. A ray of light is incident on one face of a glass prism and emerges out from the other face. Trace the path of the ray and derive an expression for refractive index of the glass prism.

54. Derive with the help of a ray diagram, an expression relating object distance, image distance and radius of curvature for a convex surface, when a ray of light travels from rarer to denser medium.

55. A ray of light is incident on one face of a prism and emerges out from the other face. Draw a ray diagram and derive an expression for refractive index of the material of the prism.

56. The image obtained with a convex lens is erect and its length is four times the length of the object. If the focal length of the lens is 20 cm, calculate the object and image distances.

57. A convex lens is used to obtain a magnified image of an object on a screen 10 cm from the lens. If the magnification is 19, find the focal length of the lens.

58. (a) How is the focal length of a spherical mirror affected when the wavelength of the light used is increased?

(b) A convex lens has 20 cm focal length in air. What is its focal length in water? (Refractive index of air-water = 1.33, refractive index of air-glass = 1.5).

59. (a) How is the focal length of a spherical mirror affected when it is immersed in water?

(b) A convex lens has 10 cm focal length in air. What is its focal length in water? (Refractive index of air-water = 1.33, refractive index of air-glass = 1.5).

60. (a) How is the focal length of a spherical mirror affected when it is immersed in glycerin?

(b) A convex lens has 15 cm focal length in air. What is its focal length in water? (Refractive index of air-water = 1.33, refractive index of air-glass = 1.5).

61. An object of 3 cm height is placed at a distance of 60 cm from a convex mirror of focal length 30 cm. Find the nature, position and size of the image formed.

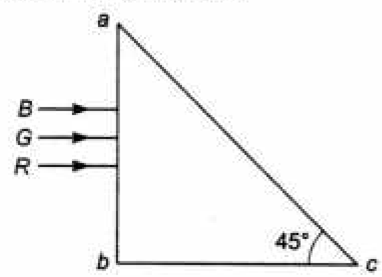
62. An object of 2 cm height is placed at a distance of 30 cm from a convex mirror of focal length 15 cm. Find the nature, position and size of the image formed.

63. A convex lens of focal length 10 cm is placed coaxially 5 cm away from a concave lens of focal length 10 cm. If an object is placed 30 cm in front of the convex lens, find the position of the final image formed by the combined system.

64. (a) With the help of a suitable ray diagram, derive the mirror formula for a concave mirror.

(b) The near point of a hypermetropic person is 50 cm from the eye. What is the power of the lens required to enable the person to read clearly a book held at 25 cm from the eye?

65. Three light rays red (*R*) green (*G*) and blue (*B*) are incident on a right angled prism *abc* at face *ab*. The refractive indices of the material of the prism for red, green and blue wavelengths are 1.39, 1.44 and 1.47 respectively. Out of the three, which colour ray will emerge out of face *ac*? Justify your answer. Trace the path of these rays after passing through face *ab*.



66. State the conditions under which total internal reflection occurs. One face of a prism with a refracting angle of 30° is coated with silver. A ray incident on another face at an angle of 45° is refracted and reflected from the silver coated face and retraces its path. Find the refractive index of the material of the prism.

67. A tank is filled with water to a height of 12.4 cm. The apparent depth of a needle lying at the bottom of the tank is measured by a microscope to be 9.3 cm. What is the refractive index of water? If water is replaced by a liquid of refractive index 1.6

upto the same height, by what distance would the microscope have to be moved to focus on the needle again?

68. The far point of a myopic person is 80 cm in front of the eye. What is the power of the lens required to enable him to see very distant objects clearly? In what way does the corrective lens help the above person? Does the lens magnify very distant objects? Explain.

69. The velocity of a certain monochromatic light, in a given transparent medium, is 2.225×10^8 m/s. What is the

(a) critical angle of incidence,

(b) polarizing angle for this medium?

70. When a ray of light passes through a triangular glass prism, find out the relation for the total deviation δ in terms of the angle of incidence i and angle of emergence e. Plot a graph showing the variation of angle of deviation with the angle of incidence and obtain the condition for the angle of minimum deviation.

71. Light incident at an angle of incidence of 45° in a certain medium, goes grazing along its surface of separation from air after refraction.

What is

(a) the velocity of light in this medium?

(b) the angle of incidence, at which light from air must be incidence on this medium, so that the refracted ray is normal to the reflected ray? Also, name the special property exhibited by light for this angle of incidence.

72. An equiconvex lens with radii of curvature of magnitude 10 cm each, is put over a liquid layer poured on top of a plane mirror. A small needle with its tip on the principal axis of the lens, is moved along the axis until its inverted real image coincides with the needle itself. The distance of the needle, from the lens, is measured to be 15 cm.

On removing the liquid layer and repeating the experiment, the distance is measured to be 10 cm.

Given that the two values of the distance measure represent the focal length values in the two cases, calculate the refractive index of the liquid.

73. (a) A ray of light falls on a triangular glass prism in such a way that the deviation of the emergent ray is minimum for that prism. Draw the ray diagram for this case and write the relation between the angle of incidence and angle of emergence.

(b) A ray of light falls on a transparent right-angled isosceles prism made from a glass of refractive index $\sqrt{2}$. Draw the ray diagram for this prism when the incident ray falls normally on one of the equal sides of this prism.

74. A double convex lens of glass of refractive index 1.5 has its both surfaces of equal radii of curvature of 20 cm each. An object of height 5 cm is placed at a distance of 15 cm from the lens. Calculate the size of the image formed.

75. How does the frequency of a beam of ultraviolet light get affected when it goes from air into glass? A ray of light incident on an equilateral glass prism shows minimum deviation of 30°. Calculate the speed of through the glass prism.

76. A beam of light converges to a point P. A lens is placed in the path of the convergent beam 12 cm from P. At what point does the beam converge if the lens is

(a) a convex lens of focal length 20 cm?

(b) a concave lens of focal length 16 cm?

Do the required calculations.

77. A convex lens made up of glass of refractive index 1.5 is dipped, in turn, in :

(a) medium A of refractive index 1.65

(b) medium B of refractive index 1.33

Explain, giving reasons, whether it will behave as a converging lens or diverging lens in each of these two media.

5 Marks Questions

78. (a) Draw a ray diagram to show refraction of a ray of monochromatic light passing through a glass prism. Deduce the expression for the refractive index of glass in terms of angle of prism and angle of minimum deviation.

(b) Explain briefly how the phenomenon of total internal reflection is used in fibre optics.

79. (a) Obtain lens maker's formula using the expression

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{(n_2 - n_1)}{R}$$

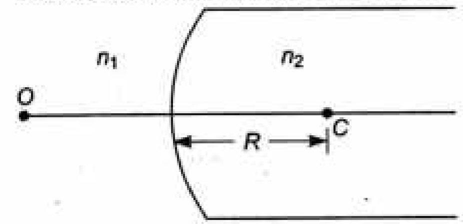
Here, the ray of light propagating from a rarer medium of refractive index (n_1) to a denser medium of refractive index (n_2) is incident on the convex side of spherical refracting surface of radius of curvature R.

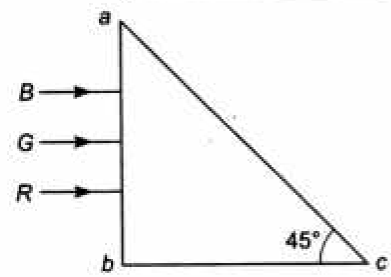
(b) Draw a ray diagram to show that image formation by a concave mirror when the object is kept between its focus and the pole. Using this diagram, derive the magnification formula for the image formed.

80. (a) A ray of monochromatic light is incident on one of the faces of an equilateral triangular prism of refracting angle A. Trace the path of ray passing through the prism. Hence, derive an expression for the refractive index of the material of the prism in terms of the angle of minimum deviation and its refracting angle.

(b) Three light rays red (R), green (G) and blue (B) are incident on the right angled prism abc at face ab. The refractive indices of the materials of the prism for red, green and blue wavelengths are respectively 1.39, 1.44 and 1.47. Trace the paths of these rays reasoning out the difference in their behavior.

81. (a) Figure shows a convex spherical surface with centre of curvature C, separating the two media of refractive indices n_1 and n_2. Draw a ray diagram showing the formation of the image of a point object O lying on the principal axis. Derive the relationship between the object and image distance in terms of refractive indices of the media and the radius of curvature R of the surface.





(b) Define the magnifying power of a compound microscope. Why should both the objective and the eyepiece have small focal lengths in a microscope?

82. (a) Derive the mirror equation using the ray diagram for the formation of a real image by a concave mirror.

(b) Define the resolving power of a telescope. Write any two advantages of a reflecting telescope over a refracting telescope.

83. Trace the rays of light showing the formation of an image due to a point object placed on the axis of a spherical surface separating the two media of refractive indices n_1 and n_2. Establish the relation between the distance of the object, the image and the radius of curvature from the central point of the spherical surface. Hence, derive the expression of the lens maker's formula.

84. (a) Draw a ray diagram for formation of image of a point object by a thin double convex lens having radii of curvatures

R_1 and R_2 and hence, derive lens maker's formula.

(b) Define power of a lens and give its SI unit. If a convex lens of length 50 cm is placed in contact coaxially with a concave lens of focal length 20 cm, what is the power of the combination?

85. (a) Draw the ray diagram for the formation of image of an object by a convex mirror and use it (along with the sign convention) to derive the mirror formula.

(b) Use the mirror formula to show that for an object, kept between the pole and focus of a concave mirror, the image appears to be formed behind the mirror.

86. Derive the lens formula, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ for a concave lens, using necessary ray diagram. Two lens of power 10 D and − 5D are placed in contact.

(a) Calculate the power of new lens.

(b) Where should an object be held from the lens, so as to obtain a virtual image of magnifications?

87. (a) For a ray of light travelling from a denser medium of refractive index n_1 to a rarer medium of refractive index n_2, prove that $\frac{n_2}{n_1} = \sin i_c$, where i_c is the critical angle of incidence for the media.

(b) Explain with the help of a diagram, how the above principle is used for transmission of video signals using optical fibres.

88. (a) Draw ray diagram showing the geometry of formation of the image of a point object situated on the principal axis and on the convex side of a spherical surface of radius of curvature R. Taking the rays as incident from a medium of refractive index n_1 to another of refractive index n_2, show that

$$\frac{n_2}{v} - \frac{n_1}{v} = \frac{n_2 - n_1}{R}$$

where, the symbols have their usual meaning.

(b) Use the relation to obtain the (thin) lens maker's formula.

89. (a) Draw a ray diagram, showing the passage of light through a glass prism. Hence, obtain a relation between the angles of deviation, incidence and emergence and the angle of prism.

(b) Show that no ray can pass through a prism whose refracting angle A is greater than twice the critical angle for the material of the prism.

90. Derive the relation between the focal length of a convex lens in terms of the radii of curvature of the two surfaces and refractive index of its material. Write the sign conventions and two assumptions used in the derivation of this relation. A convex lens of focal length 40 cm and a concave lens of focal length − 25 cm are kept in contact with each other. What is the value of power of this combination?

91. Derive the expression for the refractive index of the material of the prism in terms of the angle of the prism and angle of minimum deviation. Use this formula to calculate the angle of minimum deviation for the equilateral triangular prism of a refractive index $\sqrt{3}$.

Solutions

1. This question can be answered by considering the lens maker's formula.
From the formula, we can identify which factor will change on changing the wavelength.

The refractive index of the material of a lens increases with the decrease in wavelength of the incident light. So, focal length will decrease with decrease in wavelength according to formula.

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Thus, when we replace red light with violet light then due to increase in wavelength the focal length of the lens will decrease. **(1)**

2. Velocity of light, $c = 3 \times 10^8$ m/s. **(1)**

3. When refractive index of lens is equal to the refractive index of liquid. **(1)**

4. From Snell's law

$$n = \frac{\sin i}{\sin r} = \frac{c}{v}$$

$\Rightarrow$ $v \propto \sin r$ for given i

$\Rightarrow$ Smaller angle of refraction, smaller the velocity of light in medium.

Velocity of light is minimum in medium *A* as angle of refraction is minimum *i.e.,* 15°. **(1)**

5. Because refractive index for a given pair of media depends on the ratio of wavelengths and velocity of light in two medium not on frequency. **(1)**

6. The refractive index of diamond is much higher than that of glass. Due to high refractive index, the critical angle for diamond air interface is low. The diamond is cut suitably so that the light entering the diamond from any face suffers multiple total internal reflections at the various surfaces. **(1)**

7. Frequency remains unchanged when light travels from one transparent medium to another transparent medium. **(1)**

8. Following are the criteria for total internal reflection :

(i) Light must pass from denser to rarer medium.

(ii) Angle of incidence must be greater than critical angle. **(1)**

9. When a lens is merged in a liquid whose refractive index is more than that of the material of lens, then nature of lens changes *i.e.,* converging lens behaves like diverging lens and *vice-versa.*

Refractive index of the material of lens is less than the refractive index of water. **(1)**

10. Combined focal length of a lens combination

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

(For two thin lens in contact)

$$f_2 = -f_1$$

(focal lengths are equal, one is convex and other is concave)

$\Rightarrow$ $\frac{1}{f} = 0 \Rightarrow f = \infty.$ **(1)**

11. Speed decreases due to decrease of wavelength of wave but energy carried by the light wave depends on the amplitude of electric field vector. **(1)**

12. When a lens immersed in a liquid disappears then $\mu_{liquid} = \mu_g = 1.45$ **(1)**

13. Refer to Ans. (8) **(1)**

14. Critical angle is the angle of incidence at which angle of refraction becomes 90°. Here in this case refractive index $\mu = \frac{1}{\sin i_c}$.

$\because$ Refractive index $\mu = \frac{c}{v} = \frac{1}{\sin i_c}$

$$\Rightarrow \quad v = c \sin i_c$$
$$= 3 \times 10^8 \times \sin 30°$$
$$= 3 \times 10^8 \times \frac{1}{2}$$
$$= 1.5 \times 10^8 \text{ m/s.}$$ **(1)**

15. Due to large scattering of visible light of smaller wavelength (blue colour) as intensity of scattered light $\propto \frac{1}{\lambda^4}$. **(1)**

16. Availability of rain drops causes refraction, dispersion and internal reflection of sun light which results in the form of rainbow and the back of the observer should be towards the sun. **(1)**

17. Resultant power of the combination

$$P = P_1 + P_2 = 6 - 2 = 4D$$

$$\therefore \quad \frac{1}{f} = 4 \Rightarrow f = \frac{1}{4}\text{m} = 25 \text{ cm.}$$ **(1)**

18. Refer to Ans. (17), (f = 50 cm) **(1)**

19. Refer to Ans. (17), (f = 40 cm). **(1)**

20. Because convex mirror forms virtual erect and smaller image of object irrespective of relative position of object from mirror and therefore its field of view is very wide. **(1)**

21. In liquid

$$\frac{1}{f} = \left(\frac{\mu_g}{\mu_s} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Given $\mu_g = \mu_s$

$$\therefore \quad \frac{1}{f} = \left(\frac{\mu_g}{\mu_g} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = 0$$

or $\quad f = \infty.$ **(1)**

22. From lens maker's formula $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ or we can write $P = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$. For same lens when only medium is changed $P \propto (\mu - 1)$

Also $\quad \mu \propto \frac{1}{\lambda \text{ (wavelength)}} \quad \therefore \quad P \propto \frac{1}{\lambda}$

Due to decrease of wavelength (red to violet) the refractive index of glass increases and hence power of lens increases. **(1)**

23. Intensity of scattered light $\propto \frac{1}{\lambda^4}$ (Rayleigh criteria).

Red light have got highest wavelength in visible spectrum, therefore scatters least and hence, red light from sun able to reach on earth at the time of sunrise and sunset. Therefore the sun appears reddish at the time of sunrise and sunset. **(1)**

24.

$${}_a\mu_g = 1.5, \; {}_a\mu_w = \frac{4}{3}$$

$${}_w\mu_g = \frac{{}_a\mu_g}{{}_a\mu_w} = \frac{1.5}{4/3} = \frac{4.5}{4}$$

As $\quad \frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\therefore \quad \frac{f_2}{f_1} = \left(\frac{{}_a\mu_g - 1}{{}_w\mu_g - 1}\right)$$

$$= \frac{(1.5 - 1)}{\left(\frac{4.5}{4} - 1\right)} = \frac{0.5}{\frac{0.5}{4}} = 4$$

$$\frac{f_2}{f_1} = 4$$

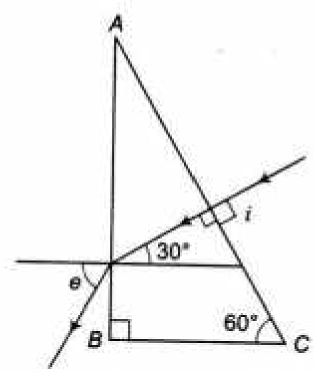
$$f_2 = 4f_1$$

Change in focal length $= 4f_1 - f_1 = f_1$ **(1)**

Change in focal length is equal to thrice of its original focal length.

25. While tracing the path of the ray, we should remember that prism bends the incident rays towards its base.

Refractive index of glass $\mu_g = \sqrt{3}$



(1)

Since, $i = 0$

At the interface AC, we have according to Snell's law

$$\frac{\sin i}{\sin r} = \frac{\mu_g}{\mu_a}$$

But $\sin i = \sin 0° = 0$

Thus, $\sin r = \frac{\mu_a \sin i}{\mu_g} = 0$

Hence, $r = 0$

This ray pass unrefracted at AC interface and reaches AB interface. Here, we can see angle of incidence becomes 30°.

Thus, applying Snell's law

$$\frac{\sin 30°}{\sin e} = \frac{\mu_a}{\mu_g} = \frac{1}{\sqrt{3}}$$

$$\sin e = \sqrt{3} \times \sin 30° = \frac{\sqrt{3}}{2}$$

Thus, $e = 60°$

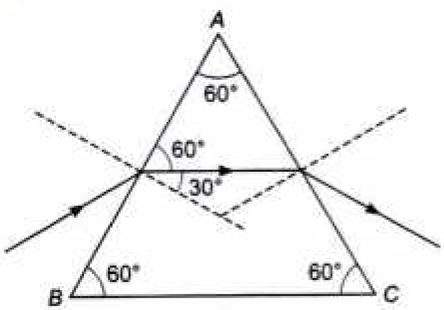
Hence, angle of emergence is 60°. **(1)**

26. To draw the ray diagram for the refraction from the prism. Following things should be kept in mind.

(i) Draw normal to the point of incidence.

(ii) Consider each boundary of the prism as separate interface and draw the ray diagram for the refraction taking place.

The reflection of light through prism is shown below.



By geometry Angle of refraction $r = 30°$ **(1/2)**

Refractive Index $\mu = \sqrt{3}$

Using Snell's law, $\mu = \frac{\sin i}{\sin r}$ **(1/2)**

$$\sin i = \mu \sin r = (\sqrt{3}) \sin(30°) = \frac{\sqrt{3}}{2}$$ **(1/2)**

Angle of incident $i = 60° = \frac{\pi}{3}$

$$i = \frac{\pi}{3}.$$ **(1/2)**

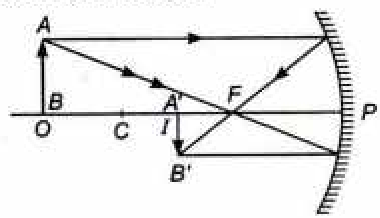
27. To draw the ray diagram for image formation, follow the rules to form the image from spherical mirror.

(i) The ray parallel to principal axis passes through the focus after reflection.

(ii) The ray passing through the focus becomes parallel to principal axis after reflection.

(iii) The ray passing through the centre of curvature returns on the same path after reflection.

(a) The ray diagram showing the image formation of the object

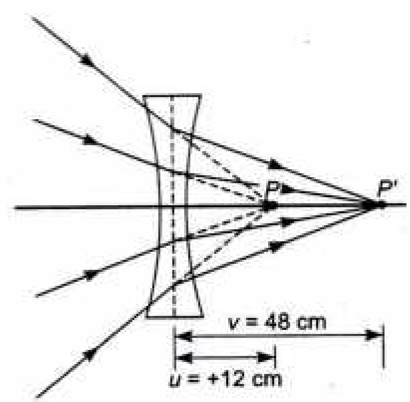


(1)

(b) The position of the image remains same whereas intensity of image reduces.

$$\left(\frac{1}{2} + \frac{1}{2} = 1\right)$$

28.



(1)

$u = +12$ cm, $f = -16$ cm, $v = ?$

Using lens equation

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$-\frac{1}{16} = \frac{1}{v} - \frac{1}{12}$$

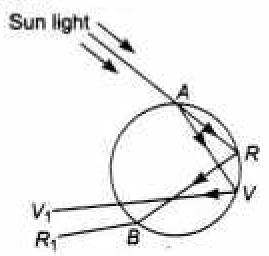
$$\Rightarrow \quad \frac{1}{v} = \frac{1}{12} - \frac{1}{16}$$

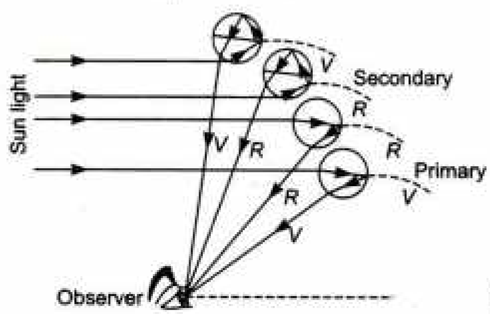
$$= \frac{4-3}{48} = \frac{1}{48}$$

$$v = +48 \text{ cm}$$

The image of virtual object at P forms at P' at a distance 48 cm from the lens. **(1)**

29. Ray diagram for the formation of rainbow is shown below.





(1)

The primary rainbow is formed by those rays which suffer one total internal reflection and two refraction and comes out of the rain drop at angle of minimum deviation.
The violet and red light colour emerge at 41° and 43° respectively and can be viewed at these angles by observer. **(1)**

30. Given,

$R_1 = +10$ cm, $R_2 = -15$ cm, $f = +12$ cm, $\mu = ?$
Lens maker's formula **(1)**

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{12} = (\mu - 1)\left(\frac{1}{10} + \frac{1}{15}\right)$$

$$= (\mu - 1)\left(\frac{5}{30}\right)$$

$$\Rightarrow \quad \mu - 1 = \frac{1}{2}$$

$$\mu = \frac{3}{2}.$$ **(1)**

31. (a) Refer to Ans (15). **(1)**

(b) Violet colour can be seen at bottom as it undergo largest deviation in visible spectrum because of highest value μ of prism for this colour being of smaller wavelength. **(1)**

32. Give, $f = \frac{2}{3}R$, $R_1 = +R$, $R_2 = -R$

$\because$ Using Lens maker's formula

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{\frac{2R}{3}} = (\mu - 1)\left(\frac{1}{R} + \frac{1}{R_1}\right)$$ **(1)**

$$\frac{3}{2R} = (\mu - 1)\left(\frac{2}{R}\right)$$

$$\mu - 1 = \frac{3}{4}$$

$$\mu = 1 + \frac{3}{4}$$

$$\mu = \frac{7}{4}.$$ **(1)**

33. (a) Refer to Ans. (23) **(1)**

(b) Refractive index μ of prism is maximum for violet and minimum for red colour. **(1)**

34. For a plano convex lens

$$R_1 = \infty$$
$$R_2 = -R$$
$$f = 0.3 \text{ m} = 30 \text{ cm}$$
$$\mu = 1.5$$
$$R = ?$$

Lens maker's formula

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$ **(1)**

$$\frac{1}{30} = (\mu - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right)$$

$$\frac{1}{30} = \frac{(1.5-1)}{R}$$

$$R = 15 \text{ cm}.$$ **(1)**

35. (a) Blue light suffers more deviation by a prism, then red light.
This happens due to high value of refractive index of material of prism for blue light because of its smaller wavelength in visible spectrum. **(1)**

(b) $\delta_m = (\mu - 1) A$

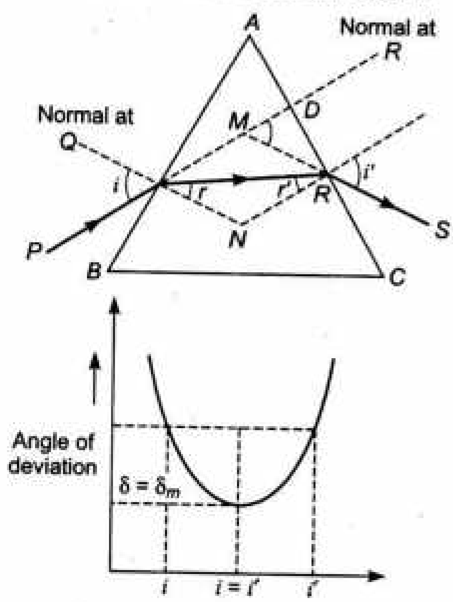
where, δ_m = angle of minimum deviation

μ = refractive index

A = prism angle. **(1)**

36. (a) $i = 52°$.
When prism is adjusted at angle of minimum deviation, then angle of incidence is equal to angle of emergence. **(1)**

(b) The ray diagram in the condition of minimum deviation is shown below.



(1)

37. (a) Optical fibre works on the principle of total internal reflection.
When a light ray travelling from denser to a rarer medium is incident at an angle greater than the critical angle then it in reflected back into the same medium. This phenomena is called total internal reflection. **(1)**

(b) Refer to Ans. (8) **(1)**

38. (a) Relation between critical angle and refractive index of a material is $\mu = \frac{1}{\sin i_c}$

where i_c = critical angle

μ = refractive index of denser medium w.r.t. rarer medium. **(1)**

(b) Yes, critical angle depend on colour of light because colour of light associated with wavelength.

Smaller the wavelength, higher the refractive index and lower the critical angle and *vice-versa*. **(1)**

39. When prism is adjusted at angle of minimum deviation, then

$$\mu = \frac{\sin i}{\sin r}$$

where, $r = \frac{A}{2} = \frac{60°}{2} = 30°$

and $i = \frac{3}{4} A = \frac{3}{4} \times 60° = 45°$

$$\Rightarrow \quad \mu = \frac{\sin 45°}{\sin 30°} = \frac{1/\sqrt{2}}{(1/2)} = \sqrt{2}$$

But refractive index **(1)**

$$\mu = \sqrt{2} = \frac{c}{v}$$

$$\Rightarrow \quad v = \frac{c}{\sqrt{2}}$$

$$= \frac{3 \times 10^8}{\sqrt{2}} = 2.1 \times 10^8 \text{ m/s}$$ **(1)**

40. For real image, magnification is taken as negative and it is equal to the ratio of image distance and object distance from the optical centre of the lens.

Given, $f = -10$ cm, magnification $m = 2$

Magnification $m = -\frac{v}{u} = 2$

$\Rightarrow \quad v = -2u$

Using mirror formula

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

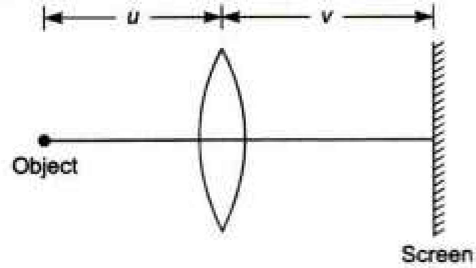
$$-\frac{1}{10} = \frac{1}{-2u} + \frac{1}{u} = \frac{1}{2u}$$

$$2u = -10$$

$$u = -5 \text{ cm}$$ **(1)**

The object must be at 5 cm from concave mirror. **(1)**

41. ∵ Object and screen are 100 cm from each other



∵ $v + u = 100$ cm ...(i)

∵ Due to reversible image of light.

$v - u = 20$ cm ...(ii)

From Eqs. (i) and (ii), we get

$v = 60, u = 40$

Now, $u = -40$ cm, $v = +60$ cm **(1)**

Using lens formula

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$= \frac{1}{60} - \left(\frac{1}{-40}\right) = \frac{1}{60} + \frac{1}{40}$$

$$\frac{1}{f} = \frac{2+3}{120}$$

$$f = \frac{120}{5}$$

$$f = 24 \text{ cm}$$ **(1)**

42. To calculate the size of the image we have to apply magnification formula.

Given, $R_1 = 30$ cm, $R_2 = -30$ cm, $\mu = 1.5$

Using Lens maker's formula

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{30} + \frac{1}{30}\right)$$

$$f = \frac{15}{0.5}$$

$$f = 30 \text{ cm}$$

Given, $u = -12.5$ cm

Using lens equation

$$\therefore \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{30} = \frac{1}{v} + \frac{1}{12.5}$$

$$\frac{1}{v} = \frac{1}{30} - \frac{1}{12.5}$$

$$\frac{1}{v} = \frac{12.5 - 30}{30 \times 12.5}$$

$$= -\frac{17.5}{30 \times 12.5}$$ **(1)**

$$v = -\frac{30 \times 12.5}{17.5}$$

$$v = -\frac{150}{7} \text{ cm}$$

But magnification $(m) = \frac{I}{O} = \frac{v}{u}$

or $$I = \frac{v}{u} \times O = \frac{\left(-\frac{150}{7}\right)}{-12.5} \times 5$$

$$I = \frac{60}{7} = 8.55 \text{ cm}$$ **(1)**

43. Refer to Ans. (24) **(2)**

44. When a lens is immersed in water then except refractive index of the material of the lens, every other factor remains unchanged. So, ratio of focal lengths will be equal to the ratio of refractive index.

Given, $R_1 = -R$, $R_2 = +R$, ${}_a\mu_g = 1.6$

Using lens maker's formula

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_a} = ({}_a\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_w} = ({}_w\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\because \quad \frac{f_w}{f_a} = \frac{({}_a\mu_g - 1)}{({}_w\mu_g - 1)} = \frac{{}_a\mu_g - 1}{\left(\frac{{}_a\mu_g}{{}_a\mu_w} - 1\right)}$$ **(1)**

$$\frac{f_w}{f_a} = \frac{(1.6 - 1)}{\left(\frac{1.6}{1.4} - 1\right)} = \frac{0.6 \times 1.4}{0.2} = 4.2$$

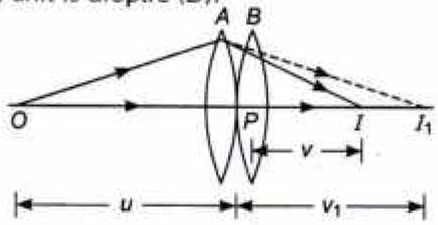
$$f_w = 4.2 f_a$$

Focal length of lens becomes 4.2 times the focal length of lens in air. **(1)**

45. The power of a lens is equal to the reciprocal of its focal length when it is measured in metre.

Power of a lens $P = \frac{1}{f(\text{metre})}$

It unit is dioptre (D).



(1/2)

Consider two lenses A and B of focal length f_1 and f_2 placed in contact with each other. An object is placed at a point O beyond the focus of the first lens A. The first lens produces an image at I_1 (real image), which serves as a virtual object for the second lens B, producing the final image at I. **(1/2)**

Since, the lenses are thin, we assume the optical centres (P) of the lenses to be coincident. For the image formed by the first lens A, we obtain

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1} \quad \text{...(i)}$$

For the image formed by the second lens B, we obtain

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2} \quad \text{...(ii)}$$

Adding Eqs. (i) and (ii), we obtain

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2} \quad \text{...(iii)} \quad \textbf{(1)}$$

If the two lens system is regarded as equivalent to a single lens of focal length f, we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \text{...(iv)}$$

From Eq. (iii) and Eq. (iv), we obtain

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f}. \quad \textbf{(1)}$$

46. For lens L_1

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \quad \textbf{(1/2)}$$

$u = -15$ cm, $v = ?$, $f = +10$ cm

$$\frac{1}{10} = \frac{1}{v} + \frac{1}{15} \quad \textbf{(1/2)}$$

Distance of image from lens L_1

$\Rightarrow \quad v = 30$ cm

For lens L_3

$$\frac{1}{f''} = \frac{1}{v''} - \frac{1}{u''}$$

Distance of image from lenss L_3

$$v'' = 10 \text{ cm}$$

$$\frac{1}{10} = \frac{1}{10} + \frac{1}{u''}$$

$\Rightarrow \quad u'' = \infty$ **(1)**

$\Rightarrow$ The refracted rays from lens L_1 becomes parallel to principal axis. It is possible only when image formed by L_1 lies at first focus of L_2 *i.e.*, at a distance of 10 cm from L_2.

$\therefore$ Separation between L_1 and L_2

$$= 30 + 10 = 40 \text{ cm}$$

The distance between L_2 and L_3 may take any value. **(1)**

47. (a) From lens maker's formula

$$\frac{1}{f} = ({}_m\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \left(\frac{{}_a\mu_g}{{}_a\mu_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{...(i)}$$

As ${}_a\mu_g < {}_a\mu_m$ (1.65) for the first medium with refractive index 1.65. **(1)**

And ${}_a\mu_g > {}_a\mu_m$(1.33) for the second medium with refractive index 1.33.

Hence, the value of focal length f will be negative in the first medium.

So, the convex lens will behave as the diverging lens for first medium and will behave as the converging lens for the second medium as the sign of the focal length will not change in second case. **(1)**

(b) Refer to Ans. (24). **(1)**

48. (a) For a mirror $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

For concave mirror $f < 0$

Also, $u < 0$ (By sign convention)

$$\frac{1}{-|f|} = \frac{1}{v} - \frac{1}{|u|}$$

(taken u and f with sign)

$$\frac{1}{v} = \frac{1}{|u|} - \frac{1}{|f|} \quad \text{...(i)}$$

where $|2f| > |u| > |f|$

$$\Rightarrow \quad \frac{1}{v} < 0$$

$$\Rightarrow \quad v < 0$$

i.e., image forms on the same side as that of object *i.e.*, real image forms for extremum value.

Also, $|2f| > |u| > |f|$

$$\frac{1}{|2f|} < \frac{1}{|u|} < \frac{1}{|f|}$$

$$\frac{1}{|2f|} - \frac{1}{|f|} < \frac{1}{|u|} - \frac{1}{|f|} < 0$$

$$-\frac{1}{|2f|} < \frac{1}{|v|} < 0$$

$$\Rightarrow \quad v > -|2f|$$

$\Rightarrow$ Image form beyond $2f$ on the same side of object. **(1)**

(b) For convex mirror, $f > 0$

Also, $u < 0$

But $\frac{1}{f} = \frac{1}{v} + \frac{1}{u} = \frac{1}{v} - \frac{1}{|u|}$

(taking u with sign)

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{|u|}$$

For f and $|u|$ to be positive

$$\frac{1}{v} > 0$$

$$\Rightarrow \quad v > 0$$

$\Rightarrow$ Virtual image forms to that of object. **(1)**

(c) For concave mirror,

$$f < 0, u < 0$$

$$|f| > |u| > 0$$

$$\because \quad \frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$-\frac{1}{|f|} = \frac{1}{v} - \frac{1}{|u|}$$

$$\frac{1}{v} = \frac{1}{|u|} - \frac{1}{|f|}$$

$$\because \quad |u| < |f|$$

$$\Rightarrow \quad \frac{1}{|u|} > \frac{1}{|f|}$$

$$\Rightarrow \quad \frac{1}{v} > 0$$

$$v > 0$$

$\Rightarrow$ Image is formed on RHS of mirror *i.e.*, virtual image.

Also, $\frac{1}{f} = \frac{1}{|v|} - \frac{1}{|u|}$

$\Rightarrow$ For concave mirror f is negative.

$$\Rightarrow \quad \frac{1}{|v|} < \frac{1}{|u|}$$

$$\Rightarrow \quad \frac{|v|}{|u|} > 1$$

$$\Rightarrow \quad m > 1$$

Enlarged, virtual image forms on the other side of mirror. **(1)**

49. Given, $f_1 = +20$ cm, ${}_a\mu_g = 1.6$, ${}_a\mu_w = 1.3$

$$\Rightarrow \quad {}_w\mu_g = \frac{{}_a\mu_g}{{}_a\mu_w} = \frac{1.6}{1.3} \quad \textbf{(1)}$$

Using lens maker's formula (in water) for converging lens

$$\frac{1}{f_2} = ({}_w\mu_g - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \quad \text{...(i)}$$

In air $\frac{1}{f_1} = ({}_a\mu_g - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$ ...(ii) **(1)**

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{f_2}{f_1} = \frac{({}_a\mu_g - 1)}{({}_w\mu_g - 1)} = \frac{(1.6 - 1)}{\left(\frac{1.6}{1.3} - 1\right)} = \frac{0.6 \times 1.3}{0.3}$$

$$\frac{f_2}{f_1} = 2.6$$

$\Rightarrow$ New focal length

$$f_2 = 2.6 \times f_1 = 2.6 \times 20$$

$$f_2 = 52 \text{ cm} \quad \textbf{(1)}$$

50. For lens of focal length 10 cm.

$$f = +10 \text{ cm}, u = -30 \text{ cm}$$

Using lens formula

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \quad \frac{1}{10} = \frac{1}{v} - \frac{1}{(-30)}$$

$$\Rightarrow \quad v = 15 \text{ cm} \qquad \textbf{(1)}$$

The image formed by Ist lens acts as an virtual object for plano-concave lens.

For plano -concave lens,

$$u = +10 \text{ cm}, f = -10 \text{ cm}, v = ?$$

Using lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$-\frac{1}{10} = \frac{1}{v} - \frac{1}{10}$$

$$\frac{1}{v} = 0$$

$$\Rightarrow \quad v = \infty. \qquad \textbf{(1)}$$

The refracted ray becomes parallel to principal axis for convex lens of focal length 30 cm.

$$u = -\infty, v = ?, f = 30 \text{ cm}$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

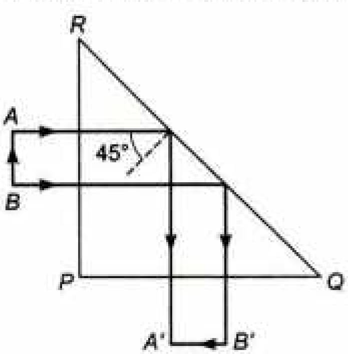
$$\Rightarrow \quad \frac{1}{30} = \frac{1}{v} - \frac{1}{(-\infty)}$$

$$\Rightarrow \quad v = 30 \text{ cm}$$

So, final image forms at a distance of 30 cm from II convex lens on the other side of it. **(1)**

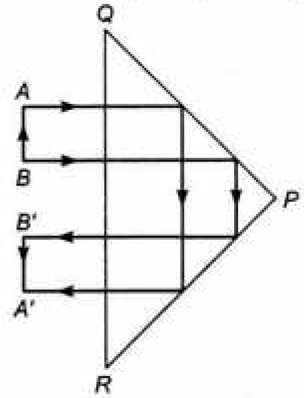
51. Refer to Ans. (8)

(a) Deviation of light rays through 90°



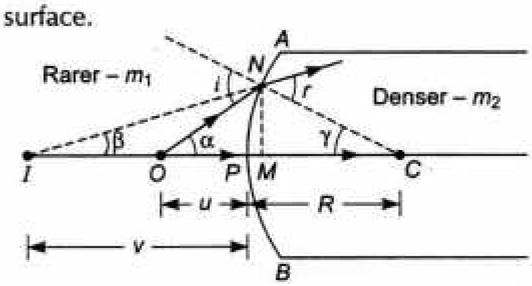
(1)

(b) Deviation of light rays through 180°



(1 × 2 = 2)

52. Let an object O is placed at a distance u from convex spherical refracting surface whose virtual image forms at I at a distance v from surface. Let R is the radius of curvature of surface.



(1)

In ΔONC, $\quad i = \alpha + \gamma \quad$ ...(i)

In ΔINC, $\quad r = \beta + \gamma \quad$ ...(ii)

Also, for small angles α, β and γ

$$\alpha \simeq \tan\alpha = \frac{NM}{OM} \simeq \frac{NM}{PO} = \frac{h}{-u}$$

(Minimum close to P)

where, $h = NM$

$$\beta \simeq \tan\beta = \frac{NM}{IM} \simeq \frac{NM}{PI} = \frac{h}{-v} \qquad \text{...(iii)}$$

Also, $\gamma \simeq \tan\gamma = \dfrac{NM}{MC} \simeq \dfrac{NM}{PC} = \dfrac{h}{+R}$ **(1)**

But by Snell's law

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$$

where, μ_2, μ_1 are the refractive indices of denser medium and rarer medium respectively.

For small angle i and r

$$\sin i \approx i, \sin r \approx r$$

$$\Rightarrow \quad \frac{i}{r} = \frac{\mu_2}{\mu_1} \Rightarrow \mu_1 i = \mu_2 r$$

$\mu_1 (\alpha + \gamma) = \mu_2 (\beta + \gamma)$ [from Eqs. (i) and (ii)]

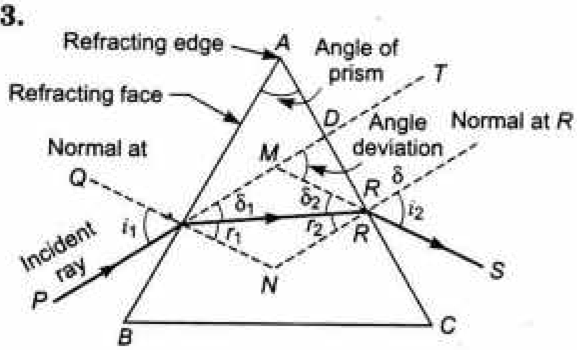
$$\mu_1 \alpha - \mu_2 \beta = \gamma(\mu_2 - \mu_1)$$

$$\mu_1 \left(\frac{h}{-u}\right) - \mu_2 \left(\frac{h}{-v}\right) = \left(\frac{h}{+R}\right)(\mu_2 - \mu_1) \text{ [from Eq. (iii)]}$$

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \quad \textbf{(1)}$$

This is the required expression.

53.



(1)

Let PQ and RS are incident and emergent rays. Let incident ray get deviated by δ by prism *i.e.,*

$$\angle TMS = \delta$$

Let δ_1 and δ_2 are deviation produced at refractors taking place at AB and AC respectively.

$$\therefore \quad \delta = \delta_1 + \delta_2$$

$$\delta = (i_1 - r_1) + (i_2 - r_2)$$

$$\delta = (i_1 + i_2) - (r_1 + r_2) \quad \text{...(i)} \quad \textbf{(1/2)}$$

Also, in quadilateral $AQNR$

$$A + \angle QNR = 180°$$

($\because$ QN and RN are normal on two surfaces)

Also, In ΔQNR,

$$\angle QNR + r_1 + r_2 = 180°$$

$$\Rightarrow \quad A = r_1 + r_2 \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\delta = (i_1 + i_2) - A \quad \text{...(iii)} \quad \textbf{(1/2)}$$

$\Rightarrow$ Angle of deviation produced by prism varies with angle of incidence. It is shown graphically below.

It is formed when prism is adjusted at angle of minimum deviation, then

$$i_1 = i_2 = i \quad \text{(say)}$$

At $\delta = \delta_m$,

$$\Rightarrow \quad r_1 = r_2 = r \quad \text{(say)}$$

From Eqs. (i) and (ii), we have

$$\delta_m = 2i - 2r$$

and $\quad 2r = A$

$$\Rightarrow \quad i = \frac{A + \delta_m}{2}$$

$$r = \frac{A}{2}$$

$\therefore$ Refractive index of material of prism is

$$\mu = \frac{\sin i}{\sin r}$$

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin \frac{A}{2}} \quad \textbf{(1)}$$

This is the required expression.

54. Refer to Ans. (52). **(3)**

55. Refer to Ans. (53). **(3)**

56. Magnification $(m) = \frac{I}{O} = \frac{v}{u}$

where, O = length of object

I = length of image

Given, $f = +20$ cm

$I = 4 \times$ length of object

$$\Rightarrow \quad \frac{I}{O} = 4$$

$$\Rightarrow \quad \frac{v}{u} = 4$$

$$v = 4u \quad \textbf{(1/2)}$$

Using lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \quad \textbf{(1/2)}$$

$$\frac{1}{f} = \frac{1}{(-4u)} - \frac{1}{(-u)}$$

$$\frac{1}{f} = -\frac{1}{4u} + \frac{1}{u}$$

$$\frac{1}{20} = \frac{4-1}{4u} = \frac{3}{4u}$$

$$u = \frac{20 \times 3}{4} = 15 \text{ cm}$$

$$u = 15 \text{ cm}$$

$$v = 4u = 15 \times 4 = 60 \text{ cm}$$

Distance of the object

$$u = 15 \text{ cm}$$

Distance of the image

$$v = 60 \text{ cm}$$

The image is on the same side of the object.

$(1 \times 2 = 2)$

57. $\because$ Real and inverted image of an object can be taken on screen.

Given, $v = +10$ cm

and magnification $m = -19$, $f = ?$

$\therefore$ $$m = \frac{I}{O} = \frac{v}{u}$$

$\Rightarrow$ $$-19 = \frac{v}{u}$$

$$v = -19u$$

$\Rightarrow$ $$u = -\frac{v}{19}$$ **(1)**

Using lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{-\left(\frac{v}{19}\right)}$$

$$\frac{1}{f} = \frac{1}{v} + \frac{19}{v}$$

$$\frac{1}{f} = \frac{20}{v}$$

$\because$ $$v = 10 \text{ cm}$$

$\therefore$ $$f = \frac{1}{2} \text{ cm}$$

$$f = 0.5 \text{ cm.}$$ **(2)**

58. (a) Focal length of spherical mirror does not get affected with the increase of wavelength. **(1)**

Using lens maker's formula

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$\therefore$ $$\frac{f_w}{f_a} = \frac{({}_a\mu_g - 1)}{({}_w\mu_g - 1)}$$ **(1)**

$$\frac{f_w}{20} = \frac{1.5 - 1}{\left(\frac{1.5}{1.33} - 1\right)} = \frac{0.5 \times 1.33}{0.17}$$

$$f_w = 78.2 \text{ cm}$$ **(1)**

59. (a) No change as f of mirror depends only on its radius of curvature. **(1)**

(b) Refer to Ans. [58 (b)] $f_w = 39$ cm **(2)**

60. Refer to Ans. (59) $f = 59$ cm **(3)**

61. Length of object $(O) = +3$ cm

$$u = -60 \text{ cm}$$

$$f = +30 \text{ cm}$$

$\because$ $$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$ (mirror formula) **(1)**

$$\frac{1}{30} = \frac{1}{v} + \frac{1}{(-60)}$$

$$\frac{1}{v} = \frac{1}{30} + \frac{1}{60} = \frac{2+1}{60}$$

$\Rightarrow$ $$v = 20 \text{ cm}$$ **(1)**

$\therefore$ $$\frac{I}{O} = -\frac{v}{u}$$

$$\frac{I}{(+3)} = -\frac{(+20)}{(-60)}$$

$$I = 1 \text{cm}$$ **(1)**

So, the virtual, erect and diminished image forms on the other side of mirror.

62. $O = 2$ cm

$$u = 60 \text{ cm}$$

$$f = 30 \text{ cm}$$

Using mirror formula

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{30} = \frac{1}{v} - \frac{1}{(-60)}$$

or $$\frac{1}{v} = \frac{1}{30} - \frac{1}{60} = \frac{2-1}{60}$$

$$\frac{1}{v} = \frac{1}{60}$$

or $\quad v = 60 \text{ cm}$ **(2)**

Linear magnification

$$m = \frac{I}{O} = -\frac{v}{u}$$

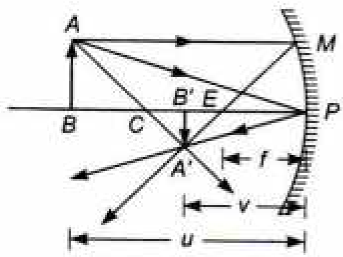
or $\quad \frac{I}{2} = -\frac{60}{(-60)}$

$$I = 2 \text{ cm}$$ **(1)**

Therefore a virtual image of 2 cm height is formed at a distance 60 cm from the mirror.

63. Refer to Ans. (50) **(3)**

64. (a)



(1)

Let an object AB is placed at a distance u from the concave mirror and its real inverted image form at a distance from it. C is centre of curvature of mirror.

Let $PB = -u$

$PB' = -v$

$PF = -f$

$CP = -R = -2f$

$\because$ ΔABC is similar to $\Delta A'B'C$

$$\Rightarrow \frac{A'B'}{AB} = \frac{CB'}{CB}$$

$$= \frac{PC - PB'}{PB - PC} = \frac{-R-(-v)}{-u-(-R)}$$

$$\frac{A'B'}{AB} = \frac{v-R}{R-u} \quad \text{...(i)}$$

Also, $\Delta A'B'P$ is similar to ΔABP

$$\therefore \frac{A'B'}{AB} = \frac{PB'}{PB}$$

$$= \frac{-v}{-u} = \frac{v}{u} \quad \text{...(ii)}$$

Comparing Eqs. (i) and (ii), we get

$$\frac{v-R}{R-u} = \frac{v}{u}$$ **(1/2)**

$$uv - uR = vR - uv$$

$$2uv = 4R + vR$$

Dividing by uvR

$$\frac{2}{R} = \frac{1}{v} + \frac{1}{u}$$

$\because$ $\quad R = 2f$

$$\Rightarrow \frac{1}{f} = \frac{1}{v} + \frac{1}{u}.$$ **(1/2)**

(b) The book be placed at a distance of 25 cm from the lens to get its image at a distance 50 cm.

$$u = -25 \text{ cm}, v = -50 \text{ cm}$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{f} = \frac{1}{-50} + \frac{1}{25} = +\frac{1}{50}$$

$$\Rightarrow f = 50 \text{ cm}$$

$$P = \frac{100}{f}$$

$$= \frac{100}{50} = 2\text{D}$$

$$P = 2\text{ D}.$$ **(1)**

65. By geometry, angle of incidence (i) of all three rays is 45°. Light suffer total internal reflection for which this angle of incidence is greater than critical angle.

$$i > i_c$$

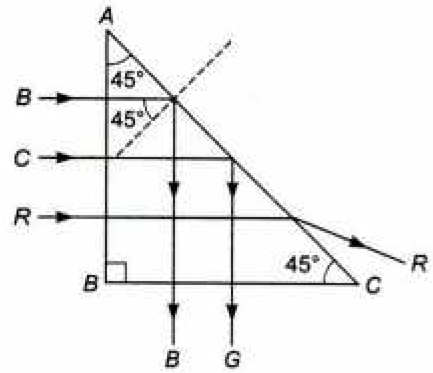
$\Rightarrow$ $\quad \sin i > \sin i_c$

or $\quad \sin 45° > \sin i_c$

or $\quad \frac{1}{\sin 45°} < \frac{1}{\sin i_c}$

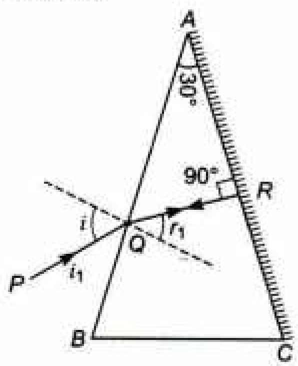
$$\sqrt{2} < \mu$$ **(1)**

Total internal reflection takes place on AC for rays $\mu > \sqrt{2} = 1.414$ *i.e.*, green and blue suffer. total internal reflection whereas red undergo refraction.



(2)

66. Refer to Ans. (8) **(1)**



(1)

The ray P retraces its path at silvered face when it falls on silvered face normally.

⇒ For II face of the prism

$$i_2 = r_2 = 0$$

But for prism,

$$r_1 + r_2 = A = 30°$$

⇒ $$r_1 = 30°$$

⇒ For I face of the prism

$$i_1 = 45°, r_1 = 30°$$

∴ $$\mu = \frac{\sin i}{\sin r} = \frac{\sin 45°}{\sin 30°} = \frac{1}{\sqrt{2}} \times 2$$

$$\mu = \sqrt{2}$$

$$\mu = 1.414$$ **(1)**

67. ∵ $$\mu = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{12.4}{9.3}$$

$$\mu = \frac{4}{3}$$ **(1)**

Refractive index of water is $\frac{4}{3}$

Again for another liquid $\mu = 1.6$

$$1.6 = \frac{12.4}{\text{Apparent depth}}$$

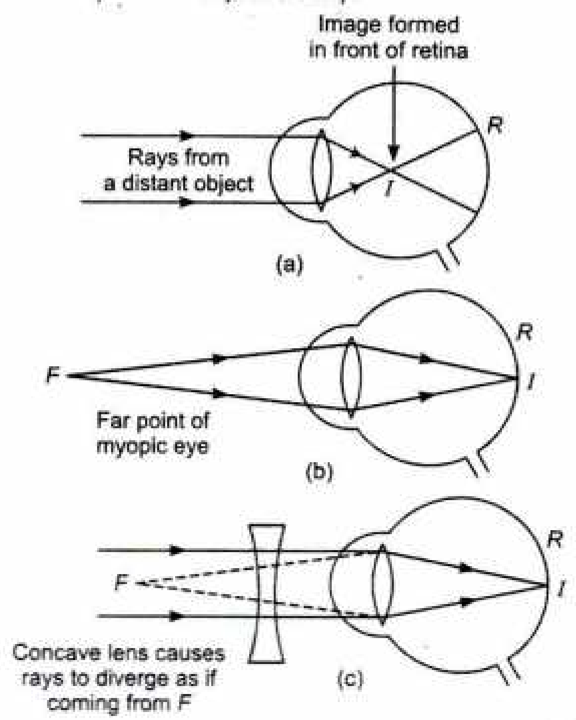
⇒ Apparent depth $= \frac{12.4}{1.6} = 7.75$ cm **(1)**

Distance moved by microscope

$= 9.3 - 7.75 = 2.18$ cm **(1)**

68. To draw the ray diagram for the myopia, we should know the correct meaning of myopia. In myopia image is formed before retina.

A concave lens of focal length f needed to see very distant object clearly.



(1)

Here, $v = -80$ cm.

As, the image of distant object must fall at far point of person.

$$u = -\infty$$

∴ $$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

⇒ $$\frac{1}{f} = \frac{1}{-80} - \frac{1}{(-\infty)}$$ **(1)**

⇒ $f = -80$ cm

⇒ $P = -1.25$ D

The corrective concave lens diverges the parallel rays from distant object as if they are coming from far point F. **(1)**

No, lens does not magnify the object.

69. (a) Given, velocity of a certain monochromatic light = 2.225×10^8 m/s

$$\sin i_c = \frac{1}{\mu} = \frac{1}{(c/v)} = \frac{v}{c} \quad \textbf{(1/2)}$$

$$\sin i_c = \frac{2.225 \times 10^8}{3 \times 10^8} = 0.7416$$

$$\sin i_c = 0.7416$$

$$i_c = \sin^{-1}(0.7416) \quad \textbf{(1)}$$

(b) $$\mu = \frac{c}{v} = \tan i_p$$

$$\frac{3 \times 10^8}{2.225 \times 10^8} = \tan i_p \quad \textbf{(1/2)}$$

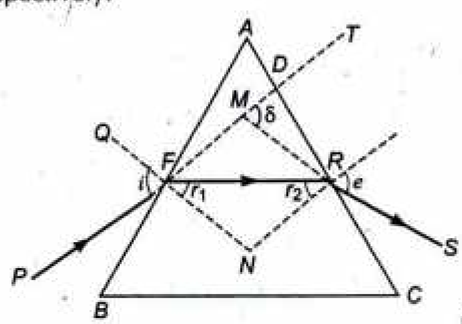
$$\tan i_p = 1.34$$

$$\Rightarrow \quad i_p = \tan^{-1}(1.34) \quad \textbf{(1)}$$

70. Let PQ and RS are incident and emergent rays. Let incident ray get deviated by δ by prism

i.e., $\angle TMS = \delta$

Let δ_1 and δ_2 are deviation produced at refractions taking place at AB and AC respectively.



(1)

$$\therefore \quad \delta = \delta_1 + \delta_2$$
$$= (i - r_1) + (e - r_2)$$
$$= (i + e) - (r_1 + r_2) \quad \ldots(i)$$

But in ΔFNR,

$$\angle FNR + \angle RFN + \angle FRN = 180°$$

or $$\angle FNR = 180° - (r_1 - r_2) \quad \ldots(ii)$$

In □ $FARNF$, $\angle AFN$ and $\angle ARN$ are right angles. So,

$$\angle FNR = 180° - A \quad \ldots(iii)$$

where, A is angle of prism.

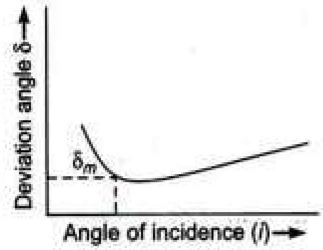
From Eqs. (i), (ii) and (iii), we have

$$A = r_1 + r_2 \quad \ldots(iv)$$

From Eqs. (ii) and (iv), we have

$$\delta = (i - e) - A \quad \ldots(v)$$

$i - \delta$ graph is shown in figure



(1)

The conditions for the angle of minimum deviation are

(i) Angle of incidence (i) and angle of emergance (e) are equal.

i.e., $\angle i = \angle e$

(ii) In equilateral prism, the refracted ray is parallel to base of prism.

(iii) The incident and emergent rays are bend on same angle from refracting surfaces of prism.

i.e., $\angle r_1 = \angle r_2$

For minimum deviation position, putting

$$r = r_1 = r_2$$

and $i = e$ in Eqs. (ii), (iv) and (v), we get

$$2r = A$$

$$r = \frac{A}{2}$$

and $$\delta_m = 2i - A$$

$$i = \frac{A + \delta_m}{2}$$

$\therefore$ Refractive index of material of prism is

$$\mu = \frac{\sin i}{\sin r}$$

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin A/2} \quad \textbf{(1)}$$

71. $i_c = 45°$

(a) By definition of critical angle *i.e.,*

$$\Rightarrow \quad \mu = \frac{1}{\sin i_c} = \frac{1}{\sin 45°} = \sqrt{2} \quad \textbf{(1/2)}$$

$$\frac{3\times10^8}{v}=\frac{\sin 45°}{\sin 30°}$$

$$=\frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}}=\sqrt{2}$$

$$v=\frac{3}{\sqrt{2}}\times10^8$$

$$=2.12\times10^8\ \text{m/s.}\qquad \textbf{(1)}$$

76. (a) $u=+12$ cm, $f=+20$ cm

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}\qquad \textbf{(1/2)}$$

$$\frac{1}{20}=\frac{1}{v}-\frac{1}{12}$$

$$\frac{1}{v}=\frac{1}{20}+\frac{1}{12}$$

$$=\frac{12+20}{240}=\frac{32}{240}$$

$$v=\frac{240}{32}=7.5\text{ cm}$$

$$v=7.5\text{ cm.}\qquad \textbf{(1)}$$

(b) $u=+12$ cm, $f=-16$ cm, $v=?$

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}\qquad \textbf{(1/2)}$$

$$\frac{1}{-16}=\frac{1}{v}-\frac{1}{12}$$

$$\frac{1}{v}=\frac{1}{12}-\frac{1}{16}$$

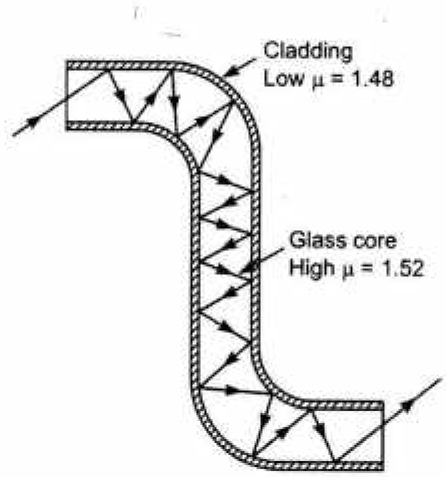
$$=\frac{4-3}{48}=\frac{1}{48}$$

$$v=48\text{ cm.}\qquad \textbf{(1)}$$

77. Refer to Ans. (47) **(3)**

78. (a) Refer to Ans. (53) **(2)**

(b) When light is incident on one end of the optical fibre at an angle of incidence greater than the critical angle for the glass cladding pair of media. The light suffers repeated total internal reflections and light travels through the optical fibre without any loss of energy from one place to other inside the optical fibre. **(2)**



(2)

79. (a) Refer to Ans. (52) **(3)**

(b) Refer Ans. 64 Eq. (ii)

$$\frac{A'B'}{AB}=\frac{PB'}{PB}=\frac{-v}{-u}$$

$$\Rightarrow\quad A'B'=-I$$

$$AB=+O$$

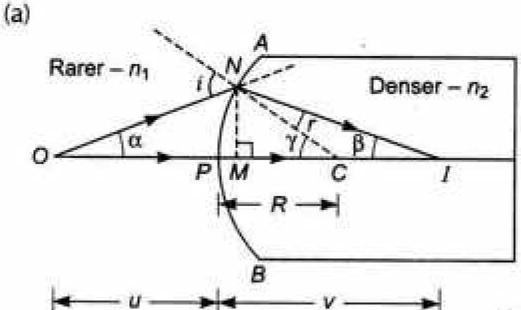
$$\Rightarrow\quad m=-\frac{I}{O}=\frac{v}{u}$$

$$m=\frac{I}{O}=-\frac{v}{u}.\qquad \textbf{(1)}$$

80. (a) Refer to Ans. (55) **(3)**

(b) Refer to Ans. (65) **(2)**

81. (a)



(1)

$$PC=+R$$

$$PI=+v$$

$$PO=-u$$

Let $NM=h$

The convex spherical refracting surface forms the image of object O at I. The radius of curvature is R

In ΔNCO, $i = \gamma + \alpha$ ...(i)

In ΔNCI, $\gamma = r + \beta$

$\Rightarrow$ $r = \gamma - \beta$ ...(ii)

For small angles α, β and γ, we have

$$\left.\begin{aligned} \alpha \simeq \tan\alpha &= \frac{MN}{MO} = \frac{MN}{PO} = \frac{+h}{-u} \\ \beta \simeq \tan\beta &= \frac{MN}{MI} = \frac{MN}{PI} = \frac{h}{-v} \\ \gamma \simeq \tan\gamma &= \frac{MN}{MC} = \frac{MN}{PC} = \frac{h}{+R} \end{aligned}\right\} \quad ...(iii)$$

Assuming M is very close to P.

By Snell's law,

$$\frac{n_2}{n_1} = \mu = \frac{\sin i}{\sin r} \quad \textbf{(1)}$$

For small i and r,

$$\frac{n_2}{n_1} = \frac{i}{r}$$

$$rn_2 = in_1$$

$$n_2(\gamma - \beta) = (\alpha + \gamma)n_1$$

[From Eqs. (i) and (ii)]

$$(n_2 - n_1)\gamma = n_1\alpha + n_2\beta$$

$$(n_2 - n_1)\left(\frac{h}{R}\right) = n_1\left(\frac{h}{-u}\right) + n_2\left(\frac{h}{v}\right)$$

[From Eq. (iii)]

$$\Rightarrow \quad \frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}. \quad \textbf{(1)}$$

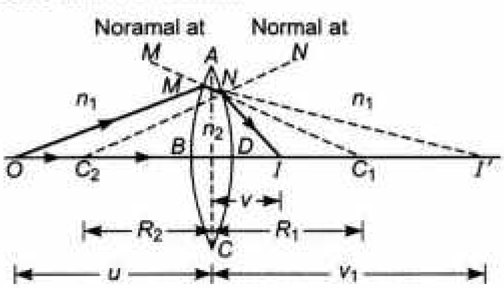
(b) Refer to Ans. 27 in topic 2. **(2)**

82. (a) Refer to Ans. (64) **(2)**

(b) Refer to Ans. (8) and Ans. (1) in topic (2). **(3)**

83. (a) Refer to Ans. [81 (a)] **(3)**

Lens maker's formula



Let I surface of lens forms the image of object O at I' which would act as virtual object for II surface and final image forms at I for refraction by I-surface.

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n-1}{R_1} \quad ...(i)$$

The image forms at I' which acts as an virtual object at I for II surface.

$$\frac{n_1}{v} - \frac{n_2}{u} = \frac{n_1 - n_2}{R_2} \quad ...(ii) \quad \textbf{(1)}$$

Adding Eqs. (i) and (ii), we have

$$\frac{n_1}{v} - \frac{n_1}{u} = (n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{v} - \frac{1}{u} = \left(\frac{n_2 - n_1}{n_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

By definition of second focal length,

$$u = -\infty, v = f$$

$$\frac{1}{f} - \frac{1}{-\infty} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \textbf{(1)}$$

This is required expression of lens maker's formula.

84. (a) Refer to Ans. 83 (b). **(2)**

(b) The reciprocal of focal length of lens is known as power of lens when focal length is taken in m.

$$P = \frac{1}{f \text{ (in metre)}}$$

SI unit of power of lens is dioptre (D) **(1)**

$$f_1 = +50 \text{ cm}, f_2 = -20 \text{ cm}$$

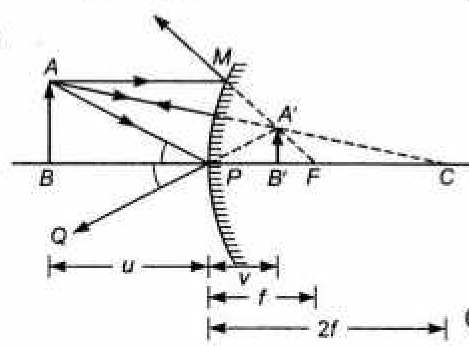
$$\because \quad \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{50} - \frac{1}{20} = \frac{2-5}{100} = \frac{-3}{100}$$

$$f = -\frac{100}{3} \text{ cm} \quad \textbf{(1)}$$

$$f = -\frac{1}{3} \text{ m} \therefore P = \frac{1}{f(\text{in m})} = \frac{1}{(-1/3)}$$

$$P = -3\text{D}. \quad \textbf{(1)}$$

85. (a)



(1)

Let convex mirror form virtual, erect diminished image $A'B'$ on the other side of mirror of an object AB as shown in above figure.

Let $PC = +2f = R$, $PB' = +v$, $PB = -u$

$\because$ $\Delta A'B'C$ is similar to ΔABC

$$\Rightarrow \quad \frac{A'B'}{AB} = \frac{CB'}{CB}$$

$$= \frac{PC - PB'}{PC + PB} = \frac{R - v}{R - u} \quad \text{...(i)}$$

Also, $\Delta A'B'P \sim \Delta ABP$.

$$\Rightarrow \quad \frac{A'B'}{AB} = \frac{AB'}{PB} = \frac{v}{-u} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{R-v}{R-u} = -\frac{v}{u}$$

$$uR - uv = -vR + uv$$

$$2uv = uR + vR \quad \text{(1)}$$

Dividing by uvR, we get

$$\frac{2}{R} = \frac{1}{v} + \frac{1}{u}$$

$\because$ $R = 2f$

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \quad \text{(1)}$$

This is required expression of mirror formula.

(b) For concave mirror, $f < 0$.

Object distanc $u < 0$ **(1)**

But $(f) > |u| > 0$

By mirror formula, we have

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$\because$ $f = -|f|$

$u = -|u|$

$$\frac{1}{v} = \frac{1}{-|f|} + \frac{1}{|u|} \Rightarrow \frac{1}{v} = \frac{1}{|u|} - \frac{1}{|f|}$$

$\Rightarrow$ $(v > 0)$

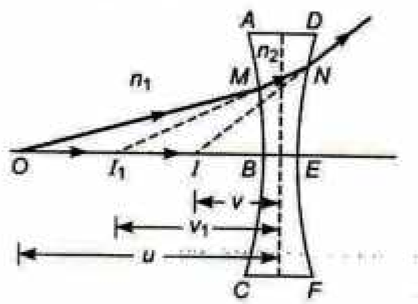
But $|u| < (f)$

$$\Rightarrow \quad \frac{1}{|u|} < \frac{1}{|f|}$$

$$\Rightarrow \quad \frac{1}{|u|} - \frac{1}{|f|} > 0 \Rightarrow \frac{1}{v} > 0. \quad \text{(1)}$$

$\Rightarrow$ The image form on the other side of mirror as v is positive.

86.



(1)

The image of object O forms at I_1 by I refractive surface of concave lens.

$$\therefore \quad \frac{n_2}{v_1} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} \quad \text{...(i)}$$

The image I_1 acts as an virtual object for II refracting surface of lens.

$$\therefore \quad \frac{n_1}{v} - \frac{n_2}{v_1} = \frac{n_1 - n_2}{R_2} \quad \text{...(ii)}$$

(1)

Adding Eqs. (i) and (ii), we have

$$n_1\left(\frac{1}{v} - \frac{1}{u}\right) = (n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{v} - \frac{1}{u} = \left(\frac{n_2 - n_1}{n_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

But by definition of second focal length

$$u = -\infty, v = f$$

$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For concave lens, R_1 is negative and R_2 is positive.

Using sign convention,

$$\frac{1}{f} = -\left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \quad \text{(1)}$$

(a) $\because$ $m = +2$

$$\therefore \quad m = \frac{v}{u} = +2$$

$$v = 2u$$

Also, $P = P_1 + P_2 = 10\,\text{D} - 5\,\text{D} = 5\,\text{D}$

(b) ∴ Focal length $= \frac{1}{P} = \frac{1}{5} = 0.2$ cm

$$f = 20 \text{ cm} \quad \textbf{(1)}$$

Applying lens formula,

$$\frac{1}{20} = \frac{1}{v} - \frac{1}{u}$$

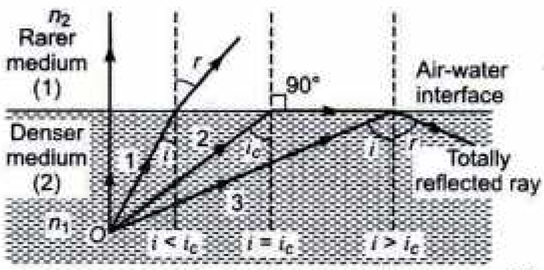
$$\frac{1}{20} = \frac{1}{2u} - \frac{1}{u} = -\frac{1}{2u}$$

$$-2u = 20$$

$$u = -10 \text{ cm}$$

The object must be placed at a distance of 10 cm from convex lens. **(1)**

87. (a)



(1)

The critical angle is that angle of incidence in denser medium for which the angle of refraction in the rarer medium is 90° as shown by ray 2. **(1)**

$${}_1n_2 = \frac{\sin i_c}{\sin 90°}$$

$$\frac{n_2}{n_1} = \sin i_c \qquad (\sin 90° = 1)$$

(b) Refer Ans. 78 (b). **(3)**

88. (a) Refer to Ans. 52 (a). **(3)**

(b) Refer to Ans. 82. **(2)**

89. (a) Refer to Ans. 53. **(3)**

(b) Refer figure and Eq. (i)

$$r_1 + r_2 = A$$

When prism is adjusted angle of minimum deviation, then

$$r_1 = r_2 = r \text{ (say)}$$

$$\Rightarrow \quad 2r = A \quad \text{...(i)}$$

For total internal reflection, critical angle on denser medium is less than r. **(1)**

$$i_c < r$$

$$2i_c < 2r$$

$$2i_c < A$$

$$\Rightarrow \quad A > 2i_c$$

⇒ If refracting angle is greater than twice of critical angle, then total internal reflection takes place and no ray comes out of the prism.

(1)

90. Refer to Ans. (52) **(3)**

$$\because \quad f_1 = 40 \text{ cm}, f_2 = -25 \text{ cm}$$

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \quad \textbf{(1)}$$

$$= \frac{1}{40} - \frac{1}{25} = \frac{5-8}{200} = \frac{-3}{200} \quad \textbf{(1)}$$

$$\Rightarrow \quad F = -\frac{200}{3} \text{ cm} \Rightarrow F = -\frac{2}{3} \text{ m}$$

⇒ Power of a lens $P = \frac{1}{f \text{ (in m)}}$

$$= -\frac{3}{2} = -1.5 \text{ D}$$

91. Refer to Ans. (53)

Refractive index **(3)**

$$\because \quad \mu = \sqrt{3},$$

Prism angle $A = 60°$ (For equilateral prism)

$$\delta_m = ?$$

Refractive index of a prism

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

where, δ_m = angle of minimum deviation.

$$\sqrt{3} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{60°}{2}\right)} \quad \textbf{(1)}$$

$$\sin\left(\frac{A+\delta_m}{2}\right) = \sqrt{3} \times \sin 30° = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \frac{A+\delta_m}{2} = \frac{\pi}{3}$$

$$\Rightarrow \quad A + \delta_m = \frac{2\pi}{3}$$

$$\text{But} \quad A = \frac{\pi}{3}$$

$$\delta_m = \frac{\pi}{3}. \quad \textbf{(1)}$$